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FROBENIUS RECIPROCITY OF DIFFERENTIABLE REPRESENTATIONS

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ABSTRACT. In this note we give the construction of the adjoint and the coadjoint of the restriction functor in the category of differentiable G-modules, where G is a Lie group.

1. **Introduction.** Let G be a Lie group, countable at infinity. A continuous representation λ of G in a complete locally convex space E is *differentiable* if for each $a \in E$ the map $\hat{a}:x \to \lambda(x)a$ of G into E is C^∞, and if the injection $a \to \hat{a}$ of E into $C^\infty(G, E)$ is a topological homeomorphism [**8**]. We then say that E is a differentiable G-module.

There is a natural way of associating a differentiable representation to any continuous, in particular unitary, representation of G. In fact, let ρ be a continuous representation of G on a complete locally convex space F. Let $F_\infty = \{a \in F: \hat{a} \in C^\infty(G, F)\}$. Then F_∞ is a dense ρ-invariant linear subspace of F. The injection $a \to \hat{a}$ sends F_∞ onto a closed subspace of $C^\infty(G, F)$. When F_∞ is equipped with the relative topology of $C^\infty(G, F)$ it becomes a complete locally convex space, and the corresponding subrepresentation λ_∞ of λ on F_∞ is differentiable. If λ is topologically irreducible then λ_∞ is topologically irreducible and conversely. For details and other basic facts concerning differentiable representations see [**8**].

The purpose of the present note is to show that the Frobenius reciprocity theorem is valid in the category of differentiable G-modules. The history of the Frobenius reciprocity theorem is long and interesting. For some recent developments the reader is referred to the work of Bruhat [**1**], Moore [**4**], Rieffel [**5**] and Rigelhof [**6**]. In particular Rigelhof succeeded in constructing an adjoint and a coadjoint for the restriction functor in the category of continuous (locally convex) G-modules.

2. **Construction of the adjoint and the coadjoint.** Let K be a closed subgroup of G, and let F be a differentiable G-module. The restriction $F \to F_K$ is a functor from the category of differentiable G-modules to the category of differentiable K-modules.

Let E be a differentiable K-module and let π be the corresponding representation.

(1) *Coadjoint functor.* Let $\mathscr{E}' = \mathscr{E}'(G)$ denote the space of distributions

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