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Twistor theory for Riemannian symmetric spaces by Francis E. Burstall and John H. Rawnsley. Springer-Verlag, Berlin and New York, 1990, 112 pp., $14.70. ISBN 3-540-52602-1

Twistor theory originated in the work of Roger Penrose in the 1960s and has developed in a number of directions. Its original context was mathematical physics, and for this aspect there are several recent surveys [9, 5, 11]. The monograph under review has very little to do with twistor theory as applied to physics and as discussed in the above references. In particular the authors state in the first sentence of their introduction: "The subject of this monograph is the interaction between real and complex homogeneous geometry and its application to the study of minimal surfaces (or harmonic maps)." The word twistor theory refers to a particular use of a twistor space associated to a Riemannian manifold in order to generate minimal surfaces or harmonic maps from holomorphic maps into the twistor space. We will say more about this below.

Weierstrass noticed in the nineteenth century that one could locally define minimal surfaces by means of holomorphic curves, i.e., any triple of holomorphic functions

$$(f_1, f_2, f_3)\colon \mathbf{C} \to \mathbf{C}^3$$

which satisfies

$$f_1^2 + f_2^2 + f_3^2 = 0,$$

determines locally a minimal surface in $\mathbf{R}^3$. This is given by the formula

$$F(z) = \operatorname{Re}\left(\int_{z_0}^{z} (f_1, f_2, f_3)\,dz\right).$$

Moreover, all minimal surfaces in $\mathbf{R}^3$ admit such a parameterization locally. This led to significant efforts in the twentieth century to represent minimal surfaces and more generally harmonic maps in terms of holomorphic objects. The monograph by Burstall and Rawnsley presents a sophisticated, elegant, and in-depth look at this question.

In the study of instantons on the 4-sphere the fibration $Z = \mathbf{P}_3(\mathbf{C}) \to S^4$ played a fundamental role in the existence theory