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ON A CONJECTURE OF FROBENIUS

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ABSTRACT. Let G be a finite group and e be a positive integer dividing the order of G. Frobenius conjectured that if the number of elements whose orders divide e equals e, then G has a subgroup of order e. We announce that the Frobenius conjecture has been proved via the classification of finite simple groups.

Let G be a finite group and e be a positive integer dividing $|G|$, the order of G. Let $L_e(G) = \{x \in G | x^e = 1\}$. In 1895 Frobenius [4] proved the following result:

$$|L_e(G)| = ke \quad \text{for an integer} \quad k \geq 1$$

and he made the following conjecture.

Frobenius conjecture. *If $k = 1$, then the e elements of $L_e(G)$ form a characteristic subgroup of G, that is, a subgroup of G that is invariant under the automorphism group of G.*

If the e elements of $L_e(G)$ form a subgroup, then $L_e(G)$ is necessarily a characteristic subgroup by the definition of $L_e(G)$. If e is a power of a prime, the conjecture is true by Sylow's theorem. M. Hall [6] gives a proof of the conjecture when G is solvable. It is proved by Zemlin [16] that the minimal counterexample to the conjecture is a nonabelian simple group. The purpose of this note is to announce the following

Theorem. *The conjecture of Frobenius is always true.*

Because of the classification of finite simple groups we may assume that G is isomorphic with

(1) A_n $(n \geq 5)$, the alternating group on n letters,
(2) a simple group of Lie type, or
(3) one of the twenty-six sporadic simple groups.

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