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GEOMETRIC AND DIFFERENTIAL PROPERTIES OF SUBANALYTIC SETS

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ABSTRACT. We announce solutions of two fundamental problems in differential analysis and real analytic geometry, on composite differentiable functions and on semicoherence of subanalytic sets. Our main theorem asserts that the problems are equivalent and gives several natural necessary and sufficient conditions in terms of semicontinuity of discrete local invariants and metric properties of a closed subanalytic set.

1. INTRODUCTION

The results announced here include solutions of two fundamental problems in differential analysis and real analytic geometry, on $\mathscr{C}^\infty$ functions composed with a proper real analytic mapping, and on formal semicoherence of subanalytic sets (a stratified real version of the coherence theory of Oka and Cartan).

The composite function problem. Let $\varphi: M \to N$ be a proper (or semiproper) real analytic mapping, and let $\varphi^*: \mathscr{C}^\infty(N) \to \mathscr{C}^\infty(M)$ denote the homomorphism of rings of $\mathscr{C}^\infty$ functions given by composition with φ. Is $\varphi^*\mathscr{C}^\infty(N)$ closed in $\mathscr{C}^\infty(M)$ (where the spaces have the $\mathscr{C}^\infty$ topology)? This problem was formulated by Thom and Glaeser [10]. It depends only on the image of φ, which is a closed subanalytic set [2, 3.5].

Subanalytic sets are the real analytic analogues of complex analytic and real semialgebraic sets, and share many of their important properties. (See, for example, [6].) But real algebraic sets already do not enjoy the coherence properties of complex analytic sets. And subanalytic sets, in general, differ in a crucial way from semialgebraic (or semianalytic) sets: The local topological dimension

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