BOOK REVIEWS

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Brauer trees of sporadic groups, by G. Hiss and K. Lux. Clarendon Press, Oxford, 1989, 525 pp., $75.00. ISBN 0-19-853381-0

Let G be a finite group. Let p be a prime, let K be a finite extension of the field of p-adic numbers, let R be the ring of integers in K, let π be a prime in R and let $\overline{R} = R/\pi R$ be the residue field of K. The theory of modular representations consists of the study of the group rings $R[G]$, $\overline{R}[G]$ and their relations with each other and with $K[G]$. It is usually convenient to take K sufficiently large so that every irreducible $K[G]$ or $\overline{R}[G]$ module is absolutely irreducible, and throughout this review it will be assumed that K has been so chosen. Since K has characteristic 0, the study of $K[G]$ is essentially equivalent to the study of $\mathbf{C}[G]$, which is due to G. Frobenius and I. Schur and is considered classical.

If V is an R free $R[G]$ module, then $V_K = V \otimes K$ is a $K[G]$ module and $\overline{V} = V \otimes \overline{R}$ is an $\overline{R}[G]$ module. This procedure can be partially reversed as follows. If X is a $K[G]$ module, then $X = V_K$ for some R-free $R[G]$ module V. The module V is far from unique; however, the composition factors of $\overline{V}$, with multiplicities, are completely determined by X. If U_1 and U_2 are $\overline{R}[G]$ modules, write $U_1 \leftrightarrow U_2$ if they have the same composition factors with multiplicities (equivalently if they are equal in the Grothendiek group defined by short exact sequences). Let $\{X_i\}$, $\{Y_u\}$ be a complete set of representatives of isomorphism classes of irreducible $K[G]$ modules, $\overline{R}[G]$ modules respectively. For each i, choose an R-free $R[G]$ module V_i with $V_i \otimes K \simeq X_i$. Let $\overline{V}_i \leftrightarrow \oplus\Sigma d_{ui} Y_u$. The remark above asserts that the d_{ui}