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NON-SELF-DUAL YANG-MILLS CONNECTIONS WITH NONZERO CHERN NUMBER

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We prove the existence of non-self-dual Yang-Mills connections on $SU(2)$ bundles over the standard four-sphere, specifically on all bundles with second Chern number not equal to ± 1. A Yang-Mills (YM) connection A is a critical point of the YM action

$$S(A) = \int_{S^4} |F|^2 d\,\mathrm{Vol} = \int_{S^4} -\mathrm{Tr}(*F \wedge F),$$

where F is the curvature of the connection A and $*$ is the Hodge dual. The YM equations $D * F = 0$, where D denotes the covariant exterior derivative, are the variational equations of this functional, and constitute a system of second-order PDE's in A. Absolute minima of the YM action, in addition to satisfying the YM equations, also satisfy a first-order system of PDE's, the (anti)self-duality equations $*F = \pm F$. We call a connection non-self-dual (NSD) if it is neither self-dual ($*F = F$) nor anti-self-dual ($*F = -F$), i.e., if it is not a minimum of the YM action.

(Anti) self-dual connections on S^4 have been well-understood for some time. The first nontrivial example, the BPST instanton [BPST], was found in 1975, and three years later all self-dual solutions on S^4 were classified [ADHM], not only for $SU(2)$ but for all classical groups. The study of self-dual $SU(2)$ connections on other four-manifolds led to spectacular progress in topology, including the discovery of fake $\mathbf{R}^4$ (see [FU] for an overview).

The study of NSD YM connections has proceeded much more slowly. While some examples of NSD YM connections on four-manifolds are known [I, Ma1, Ma2, Ur, P], until recently NSD YM connections on $SU(2)$ bundles over standard S^4 proved elusive,

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