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Cosmology in $(2+1)$*-dimensions, cyclic models, and deformations of* $M_{2,1}$, by Victor Guillemin. Ann. of Math. Stud., vol. 121, Princeton University Press, Princeton, N.J., 1989, 227 pp., $50.00 ISBN 0-691-08513-7

A Riemannian (i.e., positive definite) metric on a compact manifold is called a *Zoll* metric if all of its geodesics are simply periodic with period 2π. The classic example of a Zoll surface is S^2 with the standard metric g_s. A number of years ago, Funk proposed the problem of finding all Zoll metrics on S^2 which are close to g_s. The underlying motivation of the present monograph is to consider a generalization of this problem of Funk to Lorentzian manifolds.

A metric g on the n-dimensional manifold M is said to be *Lorentzian* if it has signature $(+, \dots, +, -)$ at all points of M. One may denote the signature of (M, g) by referring to M as a $(k+1)$-dimensional manifold where $k = n - 1$. For example, studying cosmology in $2 + 1$ dimensions may be thought of as investigating cosmological questions on three-dimensional Lorentzian manifolds.

The Levi-Civita connection ∇ and geodesics for a Lorentzian manifold (M, g) are defined in the same way as for a positive definite Riemannian manifold. In the Lorentzian case, there are three types of geodesics $\gamma\colon (a, b) \to M$ corresponding to $g(\gamma', \gamma')$ being always positive, negative, or zero. The *null* geodesics are the geodesics with $g(\gamma', \gamma') = 0$. It is an interesting fact that, up to reparameterization, the null geodesics are invariant under conformal changes. Guillemin calls a metric g on a compact manifold M a *Zollfrei* metric if all of its null geodesics are periodic. One may think of (M, g) as cyclic because each point (i.e., event) of the model gets replicated a countable number of times.