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Foliations on Riemannian manifolds, by Philippe Tondeur. Universitext, Springer-Verlag, Berlin, Heidelberg, London, New York, Paris, Tokyo, 1988, 247 pp. $29.80. ISBN 0-387-96707-9 and ISBN 3-540-96707-9. *Riemannian foliations,* by Pierre Molino, (Translated by Grant Cairns). Progress in Mathematics, vol. 73, Birkhäuser, Boston, Basel, 1986, 339 pp. $44.00. ISBN 0-8176-3370-7 and ISBN 3-7643-3370-7

Differential geometry can be described as the study of n-manifolds M having geometric structures. This ceases to be tautological when we expand on the meaning of a "geometric structure." One is given a Lie subgroup $G \subset \mathrm{Gl}(n, \mathbb{R})$, together with a coordinate atlas $\{U_\alpha, x_\alpha\}_{\alpha\in\mathfrak{A}}$, such that, on $U_\alpha \cap U_\beta$, the Jacobian $J(x_\alpha \circ x_\beta^{-1})$ of the change of coordinates is G-valued. This geometric structure, also called a G-structure, defines a reduction of the principal frame bundle $F(M)$ to the group G. Such a reduction, whether or not it comes from a G-structure, is called an "infinitesimal G-structure." If it does arise from a G-structure on M, the infinitesimal G-structure is said to be integrable.

A primary example is Riemannian geometry. This is the study of a manifold with a smoothly varying, positive definite inner product (a Riemannian metric) on its tangent spaces. In the above terminology, a Riemannian manifold is an n-manifold M, equipped with an infinitesimal $O(n)$-structure. In this case, the Riemann curvature tensor is the obstruction to integrability, so an integrable Riemannian metric (an $O(n)$-structure) is a euclidean metric on M. All manifolds admit Riemannian metrics, but very few can support euclidean geometry.

A k-dimensional foliation $\mathscr{F}$ of an n-manifold M is another example of an integrable geometric structure on M. Here the structure group G_k is the subgroup of $\mathrm{Gl}(n, \mathbb{R})$ having lower left $(n-k) \times k$ block identically zero. The coordinate atlas can be described as $\{U_\alpha, x_\alpha, y_\alpha\}_{\alpha\in\mathfrak{A}}$, where

$$x_\alpha = (x_\alpha^1, x_\alpha^2, \dots, x_\alpha^k)$$
$$y_\alpha = (y_\alpha^1, y_\alpha^2, \dots, y_\alpha^{n-k}),$$