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Initial-boundary value problems and the Navier–Stokes equations, by Heinz-Otto Kreiss and Jens Lorenz. Academic Press, New York, 1989, 398 pp., $54.50. ISBN 0-12-426125-6

When Einstein died and went to heaven, the story goes, he asked the Maker about the Grand Unified Theory. The Great Equationist unveiled a beautiful theory and soon the celestial seminar room was filled with harmonious formulae. Encouraged by such a divine answer, Einstein then asked about the Theory of Turbulence. The seminar room suddenly became dark and a thunderous voice ordered the impudent soul to leave instantly.

Allowing for poetic license at the microscopic level, the Navier–Stokes equations are nothing but Newton's laws for fluid and gases. As such, they apply universally. Neglecting thermodynamical effects, the fluid or gas is described by five functions—the three-component velocity vector field, the pressure and the density. Conservation of mass and momentum provide four equations for the unknowns. The fifth equation distinguishes between the compressible and incompressible cases. In the compressible case variations in the density induce variations in the pressure. In this case an equation of state $p = f(\rho)$ is postulated, giving the pressure as a function of the density. The function f is assumed to be known from thermodynamical considerations. Incompressibility, on the other hand, is the property that the density is constant along particle trajectories. In view of the continuity equation (conservation of mass) incompressibility is expressed in the relation $\operatorname{div} u = 0$ which is the fifth equation in this case (u is the velocity). The resulting equations are the compressible (resp., incompressible) Navier–Stokes equations if viscous effects are retained or compressible (resp., incompressible) Euler equations if these effects are neglected.

There are three fundamental questions concerning these equations. First, do solutions exist? Secondly, how do solutions depend on the physical parameters? Thirdly, what is the relevant long-time behavior of solutions?

First, the question of existence of solutions. This question has two aspects: one regarding well-posedness and the other regarding global existence. Well-posedness of an initial value problem