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COMPLETE NONCOMPACT KÄHLER MANIFOLDS WITH POSITIVE HOLOMORPHIC BISECTIONAL CURVATURE

WAN-XIONG SHI

In the theory of complex geometry and Kähler manifolds, one of the famous problems is the following conjecture:

Conjecture. Suppose M is a complete noncompact Kähler manifold with positive holomorphic bisectional curvature. Then M is biholomorphic to $\mathbf{C}^n$.

Several results concerning this conjecture were obtained in the past few years. In 1981 N. Mok, Y. T. Siu, and S. T. Yau [4] proved the following result:

Theorem 1. *Suppose M is a complete noncompact Kähler manifold of complex dimension $n \geq 2$. Suppose M is a Stein manifold and the holomorphic bisectional curvature is nonnegative. Suppose M satisfies the following assumptions:*

(i) $\mathrm{Vol}(B(x_0, r)) \geq c_0 r^{2n}$, $0 \leq r < +\infty$,

(ii) $0 \leq R(x) \leq c_1/r(x, x_0)^{2+\varepsilon}$, $x \in M$,

where $0 < c_0, c_1, \varepsilon < +\infty$ are some constants, $B(x_0, r)$ denotes the geodesic ball of radius r and centered at x_0, $\mathrm{Vol}(B(x_0, r))$ denotes the volume of $B(x_0, r)$, $r(x_0, x)$ denotes the distance between x_0 and x, and $R(x)$ denotes the scalar curvature at $x \in M$. Then M is isometrically biholomorphic to $\mathbf{C}^n$ with the flat metric.

Their result was improved by N. Mok [5] in 1984. In [5] N. Mok obtained the following result:

Theorem 2. *Suppose M is a complete noncompact n-dimensional Kähler manifold with positive holomorphic bisectional curvature and satisfies the following assumptions:*

(i) $\mathrm{Vol}(B(x_0, r)) \geq c_0 r^{2n}$, $0 \leq r < +\infty$,

(ii) $0 < R(x) \leq c_1/r(x_0, x)^2$, $x \in M$,

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