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ISOSPECTRAL FAMILIES OF CONFORMALLY EQUIVALENT RIEMANNIAN METRICS

ROBERT BROOKS AND CAROLYN GORDON

Two Riemannian manifolds are said to be isospectral if their associated Laplace operators have the same spectrum. Many examples of pairs and of continuous families of isospectral manifolds have appeared in recent years; see [Ber] for a discussion. The first examples of pairs of conformally equivalent isospectral manifolds appeared in [BT], and a general construction was given by Brooks, Perry, and Yang [BPY]. The purpose of this note is to construct continuous families of isospectral conformally equivalent manifolds, using the method of [BPY].

We begin with an example:

Let F be the Lie group of all 7×7 matrices of the form

$$\begin{pmatrix} 1 & x_1 & x_2 & z_1 & & & \\ 0 & 1 & 0 & y_1 & & & \\ 0 & 0 & 1 & y_2 & & & \\ 0 & 0 & 0 & 1 & & & \\ & & & & 1 & x_0 + x_1 & z_2 \\ & & & & 0 & 1 & y_2 \\ & & & & 0 & 0 & 1 \end{pmatrix}.$$

We denote elements of F by

$$f = f(x_0, x_1, x_2, y_1, y_2, z_1, z_2).$$

Let Γ be the integer lattice in F, that is, the set of f's where $(x_0, x_1, x_2, y_1, y_2, z_1, z_2) \in \mathbb{Z}$, and let g be any left-invariant metric on F. Let v be any smooth nonconstant periodic function on $\mathbb{R}$ of period 1, and define, for each t, $u_t \in C^\infty(F)$ by the formula

$$u_t(f) = v(x_0 + t).$$

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