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SIMPLY CONNECTED MANIFOLDS OF POSITIVE SCALAR CURVATURE

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ABSTRACT. Hitchin proved that if M is a spin manifold with positive scalar curvature, then the KO-characteristic number $\alpha(M)$ vanishes. Gromov and Lawson conjectured that for a simply connected spin manifold M of dimension ≥ 5, the vanishing of $\alpha(M)$ is sufficient for the existence of a Riemannian metric on M with positive scalar curvature. We prove this conjecture using techniques from stable homotopy theory.

Gromov and Lawson proved that any simply connected manifold of dimension ≥ 5 which is nonspin admits a metric of positive scalar curvature [GL, Corollary C]. On the other hand it was shown by Lichnerowicz that not every spin manifold has a metric of positive scalar curvature [Li]. The argument is as follows: If M is a n-dimensional spin manifold of positive scalar curvature, then by the Weizenböck formula kernel and cokernel of the Dirac operator are trivial. In particular, for $n \equiv 0 \bmod 4$ the characteristic number $\widehat{A}(M)$ which is the index of the Dirac operator vanishes. This was generalized by Hitchin, who constructed a family of Fredholm operators closely related to the Dirac operator whose index is a KO-characteristic number $\alpha(M) \in KO^{-n}(pt)$ [Hi, page 39]. Again using the Weizenböck formula he showed that $\alpha(M) = 0$ if M has a metric of positive scalar curvature. This is a strict generalization of the result of Lichnerowicz since $\alpha(M)$ can be identified with $\widehat{A}(M)$ (up to a factor) if $n \equiv 0 \bmod 4$. Gromov and Lawson proved that if M is a simply connected spin manifold of dimension ≥ 5 which is spin bordant to a manifold N of positive scalar curvature, then M admits a metric of positive scalar curvature [GL, Theorem B].

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