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A FINITENESS THEOREM FOR RICCI CURVATURE IN DIMENSION THREE

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A general problem in the study of the relations between curvature and topology lies in understanding which results concerning sectional curvature continue to hold for Ricci curvature. Recently, J. Sha and D. Yang [SY] gave examples which show that certain results such as Gromov's estimate on the Betti numbers and Cheeger–Gromoll's soul theorem, cannot be generalized to the case of Ricci curvature (compare also [AG, An]). In this note, we announce a positive result in this direction concerning finiteness theorems.

The first finiteness results were those of J. Cheeger [Ch] and A. Weinstein [We]. Cheeger's finiteness theorem is the following [Pe]:

Theorem (J. Cheeger, S. Peters). *There are only finitely many diffeomorphism types in the class of n-dimensional Riemannian manifolds satisfying*

$$|K_M| \leq \Lambda^2, \qquad \mathrm{Diam}(M) \leq D, \qquad \mathrm{Vol}(M) \geq V,$$

where K_M is the sectional curvature, $\mathrm{Diam}(M)$, *the diameter, and* $\mathrm{Vol}(M)$, *the volume of M.*

This result is recently generalized in dimensions $\neq 3, 4$ by K. Grove and P. Petersen [GP, GPW] to the following:

Theorem (Grove–Petersen–Wu). *For $n \neq 3, 4$, there are only finitely many diffeomorphism types in the class of n-dimensional Riemannian manifolds satisfying*

$$K_M \geq -\Lambda^2, \qquad \mathrm{Diam}(M) \leq D, \qquad \mathrm{Vol}(M) \geq V.$$

Along this line, it is natural to ask what happens if one replaces sectional curvature by Ricci curvature. The first attempt in answering this question is the following theorem of M. Anderson [An].

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