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THE TOPOLOGY OF COMPLETE ONE-ENDED MINIMAL SURFACES AND HEEGAARD SURFACES IN $\mathbf{R}^3$

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In this note we announce two fundamental theorems on the topological uniqueness of certain surfaces in $\mathbf{R}^3$. The first of these theorems, which will depend on the second theorem, shows that a properly embedded minimal surface in $\mathbf{R}^3$ with one end is unknotted. More precisely,

Theorem 1.1. *Two properly embedded one-ended minimal surfaces in $\mathbf{R}^3$ of the same genus are ambiently isotopic*[1].

Theorem 1.1 was conjectured by Frohman [4] who proved it in the case that the surfaces are triply periodic. A result of Callahan, Hoffman, and Meeks (Corollary 2 in [1]) states that a doubly periodic minimal surface has one end and infinite genus, when it is not a plane. Their result and Theorem 1.1 have the following corollary.

Corollary 1.1. *Any two properly embedded nonplanar minimal surfaces in $\mathbf{R}^3$ that are invariant under at least two linearly independent translations are ambiently isotopic.*

Essential in understanding the uniqueness theorems described here is the concept of a Heegaard surface in a noncompact three-manifold, which generalizes the usual notion of a Heegaard surface M in a closed three-manifold N^3. Recall that a compact embedded surface M is called a *Heegaard surface* if it separates N^3 into two genus-g handlebodies where g is the genus of M.

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[1]Two surfaces in $\mathbf{R}^3$ are ambiently isotopic if and only if there exists a one-parameter family of diffeomorphisms of $\mathbf{R}^3$ taking one surface to the other.

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