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SUBGROUPS OF POLYNOMIAL AUTOMORPHISMS

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INTRODUCTION

Throughout this paper, k will denote a commutative ring containing the rational numbers $\mathbf{Q}$, and $k^{[n]} = k[x_1, \dots, x_n]$ will be the polynomial ring over k. If $f: k^{[n]} \to k^{[n]}$ is a polynomial map (i.e., a k-algebra homomorphism), then f is a polynomial automorphism provided there is an inverse f^{-1} which is also a polynomial map. Very little is known about the group of polynomial automorphisms, and indeed it is difficult to determine which polynomial maps are automorphisms. The purpose of this paper is to define a collection H of polynomial maps, to show that each $h \in H$ is a polynomial automorphism and that H is a group under composition, and to classify H up to group isomorphism. As a consequence, this will prove a special case of the Jacobian conjecture.

If V is an n-dimensional column vector over $k^{[n]}$ whose ith term is v_i, then the Jacobian $J(V)$ is defined to be the $n \times n$ matrix $(\partial v_i/\partial x_j)$. Let X denote the vector whose ith term is x_i. If $g: k^{[n]} \to k^{[n]}$ is a polynomial map, then $g(X)$ will denote the vector whose ith term is $g(x_i)$. This gives a bijection from polynomial maps to vectors. The Jacobian $J(g)$ of a polynomial map g is defined to be the Jacobian of the vector $g(X)$. Suppose for the moment that k is a field of characteristic 0. The Jacobian conjecture states that if $f: k^{[n]} \to k^{[n]}$ is a polynomial map with $|J(f)| = 1$, then f is a polynomial automorphism. It is known that this conjecture can be reduced to the case where $J(f) = I + M$, where each term of the matrix M is a homogeneous polynomial of degree 2 [BCW]. It follows from the theorem below that the Jacobian conjecture is true, provided it can be reduced to the case where $J(f)^{-1} = I - N$, where each term of N is homogeneous of some fixed degree $q \geq 1$.

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