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THE BEHAVIOR OF THE MORDELL-WEIL GROUP OF ELLIPTIC CURVES

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1. INTRODUCTION

Suppose that E is an elliptic curve defined over $\mathbf{Q}$ given by the equation

$$y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6, \tag{1}$$

where we assume that $a_i \in \mathbf{Z}$. The set $E(\mathbf{Q})$ of solutions (x, y) with $x, y \in \mathbf{Q}$, together with the point at infinity, forms a finitely-generated abelian group, the *Mordell-Weil group* of E. It is isomorphic to $\mathbf{Z}^r \oplus F$, where F is finite and where r is the *rank* of E. The possibilities for the finite group F are completely known [9]. The important question then is to understand the behavior of the rank as E varies over elliptic curves. It is still unknown whether r is unbounded or not. In fact, one opinion is that, in general, an elliptic curve might tend to have the smallest possible rank, namely 0 or 1, compatible with the rank parity predictions of Birch and Swinnerton-Dyer [8]. We present evidence that this may not be the case.

Published examples [2, 10] of curves of rank ≥ 2 might suggest that such curves are sparsely distributed.[1] Mestre and Oesterlé found the 436 modular elliptic curves of prime conductor up to 13100, using [11]. There were 80 rank 2 curves among the 233 curves of even rank. This proportion of rank 2 curves seemed too large to conform to the conventional wisdom just stated (see also [18, pages 254–255]). We decided to investigate the ranks of elliptic curves in a systematic way, over a significantly larger range.

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[1]This situation is not unrelated to the ranks of ideal class groups of quadratic fields, where similar phenomena occur [13].

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