

ELLIPTIC METHODS IN SYMPLECTIC GEOMETRY

DUSA MCDUFF

The past few years have seen several exciting developments in the field of symplectic geometry, and a beginning has been made towards solving many important and hitherto inaccessible problems. The new techniques which have made this possible have come both from the calculus of variations and from the theory of elliptic partial differential operators. This paper describes some of the results that Gromov obtained using elliptic methods, and then shows how Floer applied these elliptic techniques to develop a new approach to Morse theory, which has important applications in the theory of 3- and 4-manifolds as well as in symplectic geometry. To give some idea of the context of their results, we begin with a section on symplectic geometry, which concentrates on questions about symplectic diffeomorphisms. For more general recent surveys of the field, see for example [A2], [E2], [G1], [G3], [H2], [V1], and [V2].

The contents of this paper are:

CONTENTS

- §1. Symplectic geometry
 - (1.1) Basic notions
 - (1.2) Symplectic images of balls
 - (1.3) Fixed point theorems
- §2 J -holomorphic spheres
 - (2.1) Basic definitions
 - (2.2) Compatible metrics and conformality
 - (2.3) The analytic setup

Received by the editors March 1, 1990.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 53C15, 58E05, 58F05.

Key words and phrases. Symplectic geometry, almost complex manifold, elliptic PDE, Hamiltonian systems, pseudoholomorphic curves.

Partially supported by NSF grant no. DMS-8803056.