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Huygens' principle and hyperbolic equations, by Paul Günther. Perspectives in Mathematics, vol. 5, Academic Press, San Diego, 1988, viii + 847 pp., $69.00. ISBN 0-12-307330-8

Each of the three prominent families of linear partial differential equations: elliptic, parabolic, and hyperbolic, has its own world of mathematical properties and corresponding physical interpretations. Perhaps the hyperbolic equations remain the most mysterious. They often describe the propagation of waves in space and time, and the physical ideas surrounding this situation are extremely powerful guides to questions such as existence and uniqueness as well as qualitative details for the solutions.

In the century where science was bursting out from the long hibernation of the Middle Age, Christian Huygens (1629–1695) put forward the wave theory of light, later to be further developed with Newton. Huygens found that such waves propagated along wave fronts, and that each point on the front acted as a point source for a new wave. Thus he could calculate actual progressing waves by superposition. In a modern formulation, this amounts to the linear hyperbolic character, in particular the finite propagation speed, of solutions to the Maxwell equations.

To explain what is meant today by Huygens' principle for a hyperbolic equation (sometimes called the strong Huygens' principle), we could simply define it to mean that sharp signals propagate as sharp signals. for example, in flat Minkowski space $\mathbf{R} \times \mathbf{R}^3$ the scalar wave equation for $u = u(t, x_1, x_2, x_3)$,

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x_1^2} - \frac{\partial^2 u}{\partial x_2^2} - \frac{\partial^2 u}{\partial x_3^2} = 0 \tag{1}$$

admits radial solutions $u = r^{-1} f(r - t)$, where $r = (x_1^2 + x_2^2 + x_3^2)^{1/2}$, and f say is smooth of compact support in $(0, \infty)$. Clearly these radial solutions propagate (in fact, they radiate away from