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HEEGAARD SURFACES IN HAKEN 3-MANIFOLDS

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The purpose of this note is to announce, for the case of Haken 3-manifolds, a complete solution of Waldhausen's conjecture concerning finiteness for Heegaard surfaces.

It is a classical result due to Moise [Moi] that all compact (orientable) 3-manifolds N have finite triangulations. Closely related to triangulations are *Heegaard graphs*, i.e., those finite, possibly disconnected, graphs Γ in N with $\Gamma \cap \partial N = \partial \Gamma$ and the property that $(N - U(\Gamma))^-$ is a handlebody (here $U(..)$ denotes the regular neighborhood in N). Indeed, the 1-skeleton of any triangulation of N is a Heegaard graph—but of course not vice versa. A surface F in N is a *Heegaard surface*, provided there is a Heegaard graph Γ in N such that F is isotopic in N to $\partial U(\Gamma \cup \partial N) - \partial N$. If F is a Heegaard surface, the pair (N, F) is called a *Heegaard splitting* for N. Heegaard splittings are independent of triangulations, and yet closely connected to them.This suggests we should use them for a classification of 3-manifolds. In this context Reidemeister [Re] and Singer [Si] showed that the existence of a common subdivision for any two triangulations implies that Heegaard surfaces in 3-manifolds are stably isotopic, i.e., isotopic modulo finite connected sums with the standard torus in S^3. Based on this observation, Waldhausen [Wa1] proved that any two Heegaard surfaces of S^3 are isotopic iff they have the same genus. Using different methods, this result has been extended to Heegaard surfaces of lens spaces [Bo, BO] and minimal (see below) Heegaard surfaces of the 3-torus [FH]. However, in [BGM] explicit examples of irreducible 3-manifolds are constructed with at least two nonhomeomorphic Heegaard splittings. In fact, it is now clear [BRZ] that uniqueness of Heegaard surfaces fails drastically and is a rather special phenomenon for 3-manifolds. In

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