by Sylvestre Gallot. But this book will clearly be *the* reference for some time to come.

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## Amenable Banach algebras, by J.-P. Pier. Pitman Research Notes in Mathematics Series, vol. 172, Longman Scientific and Technical, Harlow and New York, 1988, 161 pp., \$47.95. ISBN 0-582-01480-8

The concept of amenability was first defined for locally compact groups having evolved from the idea of a translation invariant mean or average on the bounded  $L^{\infty}$ -functions on the real line used by von Neumann. If G is a locally compact group, then (left) Haar measure m induces a left translation invariant continuous positive linear functional on  $L^1(G)$ , the space of m integrable functions. There is no such translation invariant linear functional on  $L^{\infty}(G)$ , or on several other large spaces of bounded functions, for most locally compact groups G. The groups for which there is such a positive invariant mean were called amenable by M. M. Day (1950). The transition of amenability from groups to Banach algebras arose from the transfer of Hochschild cohomology into this setting.

If X is a Banach module over a Banach algebra A, then the first (continuous Hochschild) cohomology group  $H^1(A, X)$  is the quotient of the linear space of (continuous) derivations by the space of inner derivations. A derivation D from A into X is a linear operator from A into X such that D(ab) = aD(b) + D(a)bfor all a, b in A, and D is inner if there is an x in X such that D(a) = ax - xa for all a in X. B. E. Johnson [7] showed that the amenability of a locally compact group G is equivalent to the first cohomology group  $H^1(L^1(G), X)$  being zero for each dual  $L^1(G)$ -module X. One direction of the proof uses the in-