BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 20, Number 1, January 1989

HOMOLOGY USING CHOW VARIETIES

ERIC M. FRIEDLANDER

We introduce "Lawson homology groups" $L_rH_{2r+i}(X, \mathbb{Z}_l)$ associated to an arbitrary projective algebraic variety X over an algebraically closed field k of characteristic $p \geq 0$ and a prime $l \neq p$. Our work is directly inspired by recent work of Blaine Lawson (cf. [**5**, **6**]), consisting in part of an algebraization of Lawson's geometric ideas and analytic arguments.

The Lawson homology group $L_0H_i(X, \mathbb{Z}_l)$ is the ith etale l-adic homology group of X, whereas $L_rH_{2r}(X, \mathbb{Z}_l)$ is the group of algebraic r-cycles on X modulo algebraic equivalence (see Theorem 5 below). More generally, $L_rH_{2r+i}(X, \mathbb{Z}_l)$ should be viewed as an l-adic homology group of X involving "r algebraic dimensions and i topological dimensions." As we describe below, these groups are interesting algebraic invariants with good properties which should prove useful in the study of algebraic cycles. Moreover, the author and Barry Mazur construct maps $L_rH_{2r+i}(X, \mathbb{Z}_l) \to L_{r-1}H_{2r+i}(X, \mathbb{Z}_l)$ whose iterates determine the cycle map relating algebraic cycles to etale homology.

We gratefully thank Blaine Lawson for sharing his recent results with us while still in their formative stages. We also acknowledge our great debt to Ofer Gabber whose insights were essential to our early understanding of Lawson's work. Proofs of results announced below, as well as statements and proofs of further results being developed in collaboration with Blaine Lawson and Barry Mazur, will appear elsewhere.

1. Definitions. The starting point of our work is the Chow variety $C_{r,d}(X, j)$ of effective (homogeneous, of dimension equal to) r cycles of degree d on the projective space $\mathbf{P}^N$ supported on the variety X, where X has given a closed embedding $j\colon X \subset \mathbf{P}^N$. For example, $C_{N-1,d}(\mathbf{P}^N, \mathrm{id})$ is the projective space of dimension $\binom{N+d}{d}$ whose points correspond to homogeneous forms in $N+1$ variables of degree d. Our Lawson homology groups $L_rH_{2r+i}(X, \mathbb{Z}_l)$ arise by considering the group completion of the algebraic monoid $\coprod_{d\geq 0} C_{r,d}(X, j)$ of effective r cycles on X.

We require a functor from algebraic varieties to topological spaces. We use the following composition of four functors:

$$|\cdot| = \mathrm{Re}(\cdot) \circ \mathrm{holim}(\cdot) \circ (\mathbb{Z}/l)_\infty(\cdot) \circ (\cdot)_{\mathrm{et}},$$

Received by the editors January 27, 1988.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 14F99, 14C05; Secondary 14F20.

Partially supported by the NSF.

©1989 American Mathematical Society
0273-0979/89 $1.00 + $.25 per page