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MODULI OF PARABOLIC G-BUNDLES

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Introduction. Let G be a connected reductive complex algebraic group. In this paper, the concept of parabolic structures on vector bundles [S, SM] is generalised to principal G-bundles. The relationship of parabolic bundles with unitary representations is studied and a coarse moduli space for semistable parabolic G-bundles is constructed using geometric invariant theory.

1. Let T be a maximal torus of G and let $L(T)$ be its Lie algebra. Let $L(G)$ denote the Lie algebra of G. Let E be a holomorphic principal G-bundle on a compact connected Riemann surface X. Let $E(L(G))$ denote the bundle with fibre $L(G)$ associated to E via the adjoint representation of G.

DEFINITION 1.1. Fix points $x_1, \ldots, x_n$ in X. For each i, a *parabolic structure* on E at x_i is an element $\tau_i \in L(T)$ in $E(L(G))_{x_i}$. The bundle E together with the parabolic structures $\{\tau_i\}_i$ is called a *parabolic principal G-bundle.*

DEFINITION 1.2. A parabolic G-bundle E with parabolic structures $\{\tau_i\}_i$ is *semistable* (respectively *stable*) if for every reduction $\sigma\colon X \to E/P$, P being a maximal parabolic subgroup of G, we have

$$\operatorname{degree} \sigma^*(T(G/P)) + \sum_i \overline{\mu}(\overline{\tau}_i) \geq 0 \quad (\text{resp.} > 0);$$

where $T(G/P)$ denotes the tangent bundle along the fibres of $E/P \to X$, $\overline{\mu}$ is the form on the Lie algebra $L(P)$ of P corresponding to the determinant of the adjoint action of P on $L(G)/L(P)$ and $\overline{\tau}_i$ is the conjugate of τ_i lying in $L(P)$.

We remark that the notion of a reduction to a maximum parabolic is the correct generalisation of the notion of a subbundle of a vector bundle. Giving a reduction σ as above is equivalent to giving a reduction of the structure group G to a maximum parabolic P. When $G = GL(n, C)$, $P =$ the isotropy subgroup of an r-dimensional subspace of $\mathbf{C}^n$, a G-bundle can be identified with a vector bundle E and then giving a reduction of the structure group to the maximum parabolic P is the same as giving a (proper) subbundle F of E of rank r or to be more precise, an exact sequence $0 \to F \to E \to Q \to 0$. The inequality of Definition 1.2 then

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