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ON SINGULAR HAMILTONIANS: THE EXISTENCE OF QUASI-PERIODIC SOLUTIONS AND NONLINEAR STABILITY

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In this note, we announce results concerning the existence of quasi-periodic solutions for a class of singular Hamiltonians, and the nonlinear stability of singularities (as opposed to equilibria). A third result concerns the existence of bounded, connected and open invariant sets in the neighborhood of singularities. Historically, the existence of quasi-periodic solutions and nonlinear stability of equilibria for Hamiltonian systems are established via the KAM theorems **[A1, A2, M1, M2]**. When the question of quasi-periodic behavior in the neighborhood of singularities is the issue (as in the restricted 3-body problem near one of the primaries), the singularity is transformed into an equilibrium by an appropriate regularization (cf. the classic papers **[C and M3]**). To the best of our knowledge, there is no regularization procedure for the class of singular Hamiltonians discussed here. We therefore adopt a different approach.

Existence of quasi-periodic solutions. We consider the Hamiltonian,

$$H(q_1, \ldots, q_n; p_1, \ldots, p_n) = \sum_{j=1}^{m} K_j \log |f_j(z_1, \ldots, z_n)| \tag{1}$$

with the following properties

(A) $z_j = q_j + ip_j$ and K_j are real coefficients,

(B) $f_j(z_1, \ldots, z_n)$ are entire functions of n complex variables,

(C) the zero level sets, $M_j = f_j^{-1}(0)$ are complex hyperplanes which intersect at a unique point, $\mathbf{z}^*$ (without any loss in generality, $\mathbf{z}^* = \mathbf{0}$).

This class of Hamiltonians arise in several applications notably N-body problems. In these problems $m = N(N-1)/2$, the number of edges in the complete graph consisting of N vertices; n equals $N - 1$, the number of independent degrees of freedom after the usual reduction in the presence of translational symmetry.

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