BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 20, Number 1, January 1989

PRESCRIBED HOLONOMY FOR PROJECTIVE STRUCTURES ON COMPACT SURFACES

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1. Introduction. Let S be a compact oriented surface of genus $g \geq 2$. Let U be the universal cover of S and denote the Riemann sphere by $\hat{\mathbf{C}}$. The group of 2×2 complex matrices with determinant equal to 1 will be denoted by $SL(2, \mathbf{C})$. The quotient group $PSL(2, \mathbf{C}) = SL(2, \mathbf{C})/\{\pm\mathrm{id}\}$ is canonically isomorphic to the group of conformal automorphisms of $\hat{\mathbf{C}}$.

A *projective structure* on S consists of

(i) an orientation preserving local homeomorphism (or *developing map*) $f: U \to \hat{\mathbf{C}}$, and

(ii) a homomorphism (or *holonomy map*) $\phi: \pi_1(S) \to PSL(2, \mathbf{C})$ such that $f \circ \gamma(x) = \phi(\gamma) \circ f(x)$ for all $\gamma \in \pi_1(S)$, $x \in U$.

Identifying $\hat{\mathbf{C}}$ with the boundary of the open unit ball $B^3 \subset \mathbf{R}^3$, there is a natural extension of the action of $PSL(2, \mathbf{C})$ on $\hat{\mathbf{C}}$ to an action on $B^3 \cup \hat{\mathbf{C}}$. A subgroup G of $PSL(2, \mathbf{C})$ will be called *elementary* if there is a subset of $B^3 \cup \hat{\mathbf{C}}$ consisting of one or two points which is invariant under G.

The purpose of this note is to announce

THEOREM 1. *Let $\phi: \pi_1(S) \to PSL(2, \mathbf{C})$ be a homomorphism which lifts to $SL(2, \mathbf{C})$ with $\phi(\pi_1(S))$ nonelementary. Then ϕ is the holonomy of a projective structure on S.*

It is well known that the converse is true. More precisely, if ϕ is the holonomy of a projective structure on S then

(i) $\phi(\pi_1(S))$ is nonelementary (see **[6]**), and

(ii) ϕ lifts to $SL(2, \mathbf{C})$ (see **[4,5]**).

Classically, the theory of projective structures on compact surfaces has been closely related to problems concerning uniformization and discontinuous groups. The names of Klein and Poincaré stand out in this context. In modern times the work of Ahlfors and Bers has revitalized interest in the theory and more recently the geometric, topological methods of Thurston **[8,9]** have shed new light on it. Finally, we note that Theorem 1 was conjectured by Thurston. For more details on the history of the subject see Gunning **[5]** or Hejhal **[6]** and the references therein.

In the special case that $\phi(\pi_1(S)) \subset PSL(2, \mathbf{R})$, Theorem 1 was proved by Gallo, Goldman and Porter **[1]**. If $\phi(\pi_1(S))$ has an invariant circle (for its action on $\hat{\mathbf{C}}$) then it is conjugate to a finite extension of a subgroup of $PSL(2, \mathbf{R})$. Consequently, in this case, a slight modification of the methods found in **[1]** can be used to prove the theorem.

Received by the editors January 22, 1988 and, in revised form, May 24, 1988.
1980 *Mathematics Subject Classification*. Primary 30F10; Secondary 57M99.