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TREES AND DISCRETE SUBGROUPS OF LIE GROUPS OVER LOCAL FIELDS

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Let K be a locally compact field and $\mathbf{G}$ a simple K-group, $G = \mathbf{G}(K)$. A discrete subgroup Γ of G is called a *lattice* if G/Γ carries a finite G-invariant measure. It is a *uniform* (or cocompact) lattice if G/Γ is compact and nonuniform otherwise.

When the K-rank of G is greater than one, Margulis [**Ma**, **Z**] proved that Γ is arithmetic, establishing the conjecture of Selberg and Piatetski-Shapiro. This remarkable work left open the case of rank one groups. $SL_2(\mathbb{R})$ contains continuous families of lattices (the Teichmüller spaces) and in particular it contains nonarithmetic lattices. By the Mostow rigidity theorem (cf. [**M1**]), $SL_2(\mathbb{R})$ is essentially the only real simple Lie group which allows this phenomenon. Some other real rank one groups are known to have nonarithmetic lattices: Gromov and Piatetski-Shapiro [**GPS**] showed that $SO(n, 1)$ have such lattices (earlier it was shown by Makarov and Vinberg for small n), Mostow [**M2**] constructed nonarithmetic lattices in $SU(2, 1)$, and together with Deligne [**DM**] also in $SU(3, 1)$. For the other rank one real groups: $SU(n, 1)$ ($n \geq 4$), $Sp(n, 1)$ and F_4 the problem is still open.

A related problem is the congruence subgroup problem, which asked: Given Γ arithmetic, are all its finite index subgroups congruence subgroups? Serre [**S1**] conjectured that Γ has the congruence subgroup property (CSP) if and only if $\mathrm{rank}_K(G) \geq 2$. The affirmative part of this conjecture was proved to a large extent (but mainly for nonuniform lattices; see [**R1**, **R2**] for precise results and history). Less is known for arithmetic lattices in real rank one groups: It is easy to prove that none of the arithmetic lattices in $PSL_2(\mathbb{R}) = SO(2, 1)^0$ has CSP. The same holds for $PSL_2(\mathbb{C}) = SO(3, 1)$ (see [**S1**] for the nonuniform case and [**L1**] for the uniform case). For $SO(n, 1)$, general n, it is known only for some of the lattices (Millson [**Mi**]). Similarly, Kazdhan [**Ka**] showed that some lattices in $SU(n, 1)$ do not have CSP. Again, nothing is known for $Sp(n, 1)$ and F_4, which are of rank one but have Kazdhan property (T) like groups of higher rank.

In this note we will describe results on the structure of lattices in rank one groups over locally compact *nonarchimedean* fields, as well as some

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