BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 20, Number 1, January 1989

NONLINEAR FOURIER ANALYSIS

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What is nonlinear Fourier Analysis? Let us consider an example.

Let Ω be a domain in the plane bounded by a Jordan curve Γ that goes to ∞. Let Φ be a Riemann mapping of the upper half-plane onto Ω.

How does Φ behave as a function of Γ? Is it continuous? smooth? real-analytic? Let's think of ourselves as calculus students; we are presented with this function, and we want to study its basic properties.

Of course, the Riemann mapping is not uniquely determined by the curve, and we have to do something about that. Let's ignore that issue for the moment.

There is a more serious problem with the question we've raised. To talk about the continuity of this function, we need to specify a domain of definition for it, the range space for its values, and topologies for both. To consider smoothness or real-analyticity, we need more structure on both the domain and range.

This brings us to another component of the above question: What are the most natural choices for the domain and range? We would like the domain to be as large as possible so that the function is defined and well behaved. In this case, the domain is a space of curves, and we want in particular to minimize the smoothness assumptions on the curves. We would like for the domain to be a space of curves characterized by some natural geometric condition.

There is a beautiful theorem of Coifman and Meyer **[CM3]** that says that the Riemann mapping is a real-analytic function on a natural space of curves, and with a natural choice of range space. It is a bit technical to state it precisely now, but I shall say more about it at the end of this section.

The problem of understanding the Riemann mapping as a function of the curve is a good example of a problem in nonlinear Fourier analysis. There are many basic objects in mathematics which can be viewed naturally as nonlinear functions in infinite dimensions, and we want to study their basic properties as such.

There are also cases in which we are forced to confront such issues of nonlinear dependence even when the original problem does not directly call for it. We shall see examples of this shortly.

The reason that this is nonlinear Fourier analysis, and not just nonlinear analysis, is that we often need Fourier analysis to deal with the objects that

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Received by the editors April 15, 1988.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 42B99.
The author is partially supported by the NSF and the Alfred P. Sloan Foundation.