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SOME LOCAL-GLOBAL RESULTS IN FINITE TRANSFORMATION GROUPS

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ABSTRACT. The main theorem is a converse to the localization theorem in equivariant cohomology, using the Sullivan fixed point conjecture. In applications, we prove that certain fibrations over BG are fiber homotopy equivalent to the Borel constructions of finite dimensional (finitely dominated) G-spaces.

Introduction. Let G be a finite group, and let X be a G-space. The Borel equivariant cohomology of X is denoted by

$$H_G^*(X) \equiv H^*(E_G \times_G X; \mathbf{F}_p),$$

where $E_G \times_G X \xrightarrow{\pi} BG$ is the twisted product associated to the universal bundle $E_G \to BG$. For a p-elementary abelian group, i.e.

$$G \simeq (\mathbf{Z}/p\mathbf{Z})^n, \bigoplus_{k \geq 0} H^{2k}(BG; \mathbf{F}_p)_{\mathrm{red}} \simeq \mathbf{F}_p[t_1, \ldots, t_n]$$

is a polynomial algebra denoted by H_G. Then $H_G^*(X)$ is a graded H_G-module and the localization $S^{-1}H_G^*(X)$ is obtained by inverting nonzero elements of $H^2(BG; \mathbf{F}_p)$. According to the Borel localization theorem [**B**], for a finite dimensional G-space X, the inclusion of the fixed points $j\colon X^G \to X$ induces an $S^{-1}H_G$- isomorphism $j^*\colon S^{-1}H_G^*(X) \xrightarrow{\simeq} S^{-1}H_G^*(X^G)$. Further refinements of localization in equivariant cohomology with deep applications are due to Quillen [**Q**] and W. Y. Hsiang [**Hs**], where the p-elementary abelian groups (and tori) play a key role in an algebro-geometric setting. In [**B, Hs, Q**] and related developments, the hypothesis $\dim X < \infty$ is indispensable since the localization theorem fails if $\dim X = \infty$. In [**Su**] D. Sullivan pointed to new directions. Let $\mathrm{Map}(E_G, X)$ be the mapping space together with the conjugation G-action $f^g(x) = g^{-1}f(gx)$, so that the subspace of equivariant maps $\mathrm{Map}_G(E_G, X)$ and the fixed point set $\mathrm{Map}(E_G, X)^G$ coincide. In 1970 Sullivan conjectured that for $G = \mathbf{Z}/2\mathbf{Z}$, the inclusion $X^G \to \mathrm{Map}_G(E_G, X)$ (via constant maps) induces a weak homotopy equivalence after 2-adic completion [**Su**]. In 1982, H. Miller's proof of the important special case $X = X^G$ marked a new era in transformation groups and homotopy theory [**M**]. As pointed out by H. Miller,

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