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STRUCTURE THEORY AND REFLEXIVITY OF CONTRACTION OPERATORS

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1. Introduction. Let $\mathscr{H}$ be a separable, infinite-dimensional, complex Hilbert space, and let $\mathscr{L}(\mathscr{H})$ denote the algebra of all bounded linear operators on $\mathscr{H}$. The purpose of this note is to announce several new, and rather general, sufficient conditions that a contraction T in $\mathscr{L}(\mathscr{H})$ be reflexive, and, at the same time, to give various characterizations of the class of those contractions that possess an analytic invariant subspace (definition given below). Complete proofs and other results will appear in [**7**]. The principal new idea involved is a considerable improvement of the main construction of §3 of [**9**]. The new reflexivity theorems also depend on techniques from [**9**, **3**, **1**, and **4**], and yield, in particular, the following improvement of the main result of [**4**].

THEOREM 1.1. *If T is a contraction in $\mathscr{L}(\mathscr{H})$ such that the spectrum $\sigma(T)$ of T contains the unit circle $\mathbf{T}$, then either T is reflexive or T has a nontrivial hyperinvariant subspace.*

If $T \in \mathscr{L}(\mathscr{H})$ we denote by $\mathscr{A}_T$ the dual algebra generated by T (i.e., $\mathscr{A}_T$ is the smallest unital subalgebra of $\mathscr{L}(\mathscr{H})$ containing T that is closed in the weak* topology (which accrues to $\mathscr{L}(\mathscr{H})$ by virtue of its being the dual space of the Banach space $\mathscr{C}_1(\mathscr{H})$ of trace-class operators)). It follows that $\mathscr{A}_T$ is the dual space of $Q_T = \mathscr{C}_1(\mathscr{H})/{}^{\perp}\mathscr{A}_T$, where ${}^{\perp}\mathscr{A}_T$ is the preannihilator of $\mathscr{A}_T$ in $\mathscr{C}_1(\mathscr{H})$, under the pairing

$$\langle A, [L]\rangle = \operatorname{tr}(AL), \qquad A \in \mathscr{A}_T,\ L \in \mathscr{C}_1(\mathscr{H}),$$

where $[L]$ denotes the element of the quotient space Q_T containing the trace-class operator L. Thus, if x and y are vectors in $\mathscr{H}$, then $[x \otimes y]$ denotes the element of Q_T containing the rank-one operator $x \otimes y$. The dual algebra $\mathscr{A}_T$ is said to have property $(\mathbf{A}_{1,\aleph_0})$ if for any sequence $\{[L_j]\}_{j=1}^{\infty}$ of elements from Q_T there exist vectors x and $\{y_j\}_{j=1}^{\infty}$ in $\mathscr{H}$ satisfying

$$[L_j] = [x \otimes y_j], \qquad j = 1, 2, \ldots . \tag{1}$$

If, moreover, there exists $\rho \geq 1$ (independent of the family $\{[L_j]\}$) with the property that for every $s > \rho$, the vectors $\{x\}$ and $\{y_j\}$ satisfying (1) can also be chosen to satisfy

$$\|x\| \leq \left(s \sum_{k=1}^{\infty} \|[L_k]\|\right)^{1/2}, \qquad \|y_j\| \leq (s\|[L_j]\|)^{1/2}, \qquad j = 1, 2, \ldots,$$

then we say that $\mathscr{A}_T$ has property $(\mathbf{A}_{1,\aleph_0}(\rho))$.

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