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CONSTRAINED POISSON ALGEBRAS AND STRONG HOMOTOPY REPRESENTATIONS

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A Poisson algebra is a commutative associative algebra A with an (anticommutative) bracket $\{\ ,\ \}$ which is a derivation with respect to the commutative product: $\{f, gh\} = \{f, g\}h + f\{g, h\}$. Constraints constitute a distinguished set of elements ϕ_α of A. They are said to be *first class* constraints if the ideal I they generate (under the commutative product) is closed under Poisson bracket; I need not be an ideal with respect to $\{\ ,\ \}$. This structure arises in physics with $A = C^\infty(W)$ for some symplectic manifold W. The constraints determine a subvariety $V \subset W$, the zero locus of I, and a foliation $\mathscr{F}$ of V, by the flows determined by the derivations $\{\ ,\ \}$. One wishes to compute the $\operatorname{ad} I$-invariant functions on V, which would give $C^\infty(V/\mathscr{F})$ were the foliation to give a submersion $V \to V/\mathscr{F}$ onto a manifold.

In a remarkable series of papers, Fradkin, Batalin and Vilkovisky [**0**–**3**, **6**] and then Henneaux [**10**] developed a method for calculating the $\operatorname{ad} I$-invariant functions in $C^\infty(V) = A/I$ without passing through the quotient A/I. The method appeared to depend on solving certain specific, complicated equations and initially was applicable only locally and when I was a regular ideal.

Using the techniques of 'homological perturbation theory' [**7**, **8**, **9**], I am able to justify their machinery in terms of the algebra alone, including, with Henneaux [**11**], the case of nonregular ideals [**0**]. The idea for this approach owes a great deal to the paper of Browning and McMullan [**4**], which revealed the structure of a multicomplex implicit in Fradkin et al and Henneaux.

The Lie algebra cohomology $H^0(I, A/I)$ computes the $\operatorname{ad} I$-invariant functions on V, but physics requires a description in terms of A and prefers to use Φ, the linear span of the constraints ϕ_α, rather than the full ideal I. An obvious step algebraically is to replace A/I by a free resolution over A. To combine this with the restriction to $\Phi \subset I$ is more subtle.

The Lie algebra cohomology of Cartan, Chevalley and Eilenberg [**5**] begins with the algebra $\operatorname{Alt}(I, A/I)$ of alternating multilinear functions on I with values in A/I and a differential Alt $\to$ Alt (which increases the number of variables by one) given in terms of the bracket on I and the adjoint representation of I on A/I: For example, for $h\colon I \to A$, we have

$$(\delta h)(f, g) = h(\{f, g\}) - \{f, h(g)\} + \{g, h(f)\}.$$

The subalgebra $\operatorname{Alt}_A(I, A/I)$ of A-multilinear functions is in fact a subcomplex with the same H^0. (This is isomorphic to the complex which defines

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