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STRUCTURE OF FOURIER AND FOURIER–STIELTJES COEFFICIENTS OF SERIES WITH SLOWLY VARYING CONVERGENCE MODULI

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Ostensibly, convergence problems regarding the Fourier series in L^1 parallel the classical Tauberian problems. Let $f \in L^1(T)$, $T = \mathbf{R}/2\pi\mathbf{Z}$, then the partial sums $S_n(f) = S_n(f,t) = \sum_{|k|\le n} \hat{f}(k)e^{ikt}$ are $(C,1)$-summable, both pointwise and in L^1-norm. Inasmuch as the appropriate Tauberian conditions are available, the convergence questions may be settled in the standard manner. However, Tauberian conditions needed to recover L^1-convergence originate from the Hausdorff-Young inequality and do not have a straightforward analogue in the elementary Tauberian theory. Such a condition is obtained in [**1**], i.e.

$$\lim_{\lambda\to 1+0} \overline{\lim_{n}} \sum_{|k|=n+1}^{[\lambda n]} |k|^{p-1}|\Delta \hat{f}(k)|^p = 0, \tag{1}$$

where $1 < p \le 2$ and $f \in L^1(T)$. Later in [**2** and **3**], the condition (1) has been further extended and studied. Although (1) is much weaker than the classical [**4**, **5**] and neoclassical [**6**, **7**] regularity and/or speed conditions, it does not provide explicit information about the Fourier coefficients. To overcome this shortcoming a new approach is proposed in [**8**], based on regular variation of the convergence moduli.

A nondecreasing sequence $\{R(n)\}$ of positive numbers is $*$-regularly varying if

$$\lim_{\lambda\to 1+0} \overline{\lim_{n}} \frac{R([\lambda n])}{R(n)} \le 1;$$

or more generally, the sequence $\{R(n)\}$ is O-regularly varying if

$$\overline{\lim_{n}} \frac{R([\lambda n])}{R(n)}$$

is finite for $\lambda > 1$. In particular, if $\lim_n R([\lambda n])/R(n) = 1$, $\{R(n)\}$ is slowly varying.

Let $\{c(n)\}$ be a sequence of complex numbers and let $\sum_{|n|<\infty} c(n)e^{int}$ be its formal trigonometric transform. The convergence modulo of the trigonometric transform is defined as

$$K_n^p(c) = \sum_{|k|\le n} |k|^{p-1}|\Delta c(k)|^p, \qquad p > 1.$$

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