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TOPOLOGICAL RIGIDITY FOR HYPERBOLIC MANIFOLDS

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ABSTRACT. Let M be a complete Riemannian manifold having constant sectional curvature -1 and finite volume. Let $\overline{M}$ denote its Gromov-Margulis manifold compactification and assume that the dimension of M is greater than 5. (If M is compact, then $\overline{M} = M$ and $\partial\overline{M}$ is empty.) We announce (among other results) that any homotopy equivalence $h\colon (N, \partial N) \to (\overline{M}, \partial\overline{M})$, where N is a compact manifold, is homotopic to a homeomorphism. This is a topological analogue of Mostow's rigidity theorem [**18**]. Moreover, for each integer j, the surgery group $L_j(\pi_1 M)$ is isomorphic to the set of homotopy classes of maps $[I^k \times \overline{M} \operatorname{rel} \partial, G/\mathrm{Top}]$ where k is any positive integer such that $k + \dim M \equiv j \bmod 4$. Here I^k denotes the k-fold product $I \times I \times \cdots \times I$ where I is the closed interval $[0, 1]$.

Let M denote a complete Riemannian manifold having constant sectional curvature -1 and finite volume. Thus M is a real hyperbolic manifold of finite volume. Gromov [**13**] and Margulis have constructed a smooth manifold compactification of M which is denoted by $\overline{M}$. Let I^k denote the k-fold product $I \times I \times \cdots \times I$, where I is the closed interval $[0, 1]$; in particular, I^0 is a single point. Let N be a compact manifold such that its boundary ∂N decomposes as $\partial N = \partial_1 N \cup \partial_2 N$ where $\partial_1 N, \partial_2 N$ are compact codimension zero submanifolds of ∂N with $\partial(\partial_1 N) = \partial(\partial_2 N) = \partial_1 N \cap \partial_2 N$. Set $\Lambda N = \partial(\partial_1 N)$.

THEOREM 1. *Let $h\colon (N, \partial_1 N, \partial_2 N, \Lambda N) \to (I^k \times \overline{M}, \partial I^k \times \overline{M}, I^k \times \partial\overline{M}, \partial I^k \times \partial\overline{M})$ be a homotopy equivalence of* 4*-tuples such that $h\colon \partial_1 N \to (\partial I^k) \times \overline{M}$ is a homeomorphism. If $k + \dim(M) > 5$, then there is a homotopy*

$$h_t\colon (N, \partial_1 N, \partial_2 N, \Lambda N) \to (I^k \times \overline{M}, \partial I^k \times \overline{M}, I^k \times \partial\overline{M}, \partial I^k \times \partial\overline{M}), \quad t \in [0, 1],$$

with $h_0 = h$, h_1 a homeomorphism and the restriction of h_t to $\partial_1 N$ the constant homotopy. Moreover, if the restriction of h to $\partial_2 N$ is also a homeomorphism, then we need only assume that $k + \dim(M) > 4$ and h_t can be constructed so that it is constant on all of ∂N.

COROLLARY 1. *Let $h\colon (N, \partial N) \to (\overline{M}, \partial\overline{M})$ be a homotopy equivalence of pairs where N is a compact manifold. If $\dim(M) > 5$, then there is a homotopy*

$$h_t\colon (N, \partial N) \to (\overline{M}, \partial\overline{M}), \qquad t \in [0, 1],$$

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