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Invariant manifolds, entropy and billiards; smooth maps with singularities, by Anatole Katok and Jean-Marie Strelcyn, with the collaboration of F. Ledrappier and F. Przytycki. Lecture Notes in Mathematics, vol. 1222, Springer-Verlag, Berlin, Heidelberg, New York, London, Paris, Tokyo, 1986, viii + 283 pp., $23.60. ISBN 0-387-17190-8

Many dynamical systems arising in physics, meteorology, chemistry, biology, engineering and other fields exhibit chaotic behavior. There is no precise definition of "chaos"; however, in simple terms, chaotic behavior means that a typical orbit seems to wander aimlessly in the phase space with no identifiable pattern and its future is unpredictable although the system itself is deterministic in nature. The only known cause of chaos is hyperbolicity. Suppose we begin moving along a hyperbolic orbit with the speed prescribed by the system and observing the relative motion of nearby orbits that start on a codimension 1 transversal to our orbit. Then in an appropriate coordinate system the relative motion up to first-order terms will be the same as in a neighborhood of a saddle point $\dot{x} = \Lambda x$, $\dot{y} = My$. The eigenvalues of Λ have strictly negative real parts; the eigenvalues of M have strictly positive real parts. These real parts are called Lyapunov characteristic exponents (LCEs) and give us the exponential rates with which nearby trajectories move to or away from our orbit. If all orbits are hyperbolic, all LCEs are uniformly separated from 0 and all estimates are uniform, then we have an Anosov system; a good example is the geodesic flow on a compact surface of curvature -1. D. Anosov and Ya. Sinaĭ studied such systems about 20 years ago. They constructed invariant families (or foliations) of stable and unstable manifolds and used them to prove ergodicity (i.e., chaos) for Anosov systems preserving an absolutely continuous measure. In the mid-70s Ya. Pesin generalized the whole theory for smooth nonuniformly hyperbolic dynamical systems, i.e., systems for which LCEs are not bounded away from 0 and some may actually equal 0. He followed a similar path by constructing and using the stable and unstable manifolds and proved that the measure-theoretic entropy equals $\sum_j \int \mu_j \, dm$, where the μ_j's are positive exponents and m is an absolutely continuous invariant measure. Later D. Ruelle, R. Mane and others simplified and generalized some of Pesin's results.

In their book A. Katok and J. M. Strelcyn generalize the Pesin theory for the case of a dynamical system with singularities. An example of such a system and one of the main motivations for the book is a billiard system, i.e.,