RESEARCH ANNOUNCEMENTS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 18, Number 2, April 1988

TOPOLOGICAL TRANSVERSALITY HOLDS IN ALL DIMENSIONS

FRANK QUINN

Moving one submanifold to be transverse to another is a basic and essential operation in the study of manifolds. In the differentiable and PL categories the proof that this operation is always possible is straightforward, and one of the first objectives in developments of the subject. In the topological category it is more difficult; the first progress came in the profound work of Kirby and Siebenmann [**4**]. Most of what they left unsettled was completed in [**7**], but a few resistant 4-dimensional cases have remained open. Here we sketch a proof of these exceptional cases; details will appear in Chapter 9 of [**3**]. We also indicate modifications required in the proof of [**4**] to reduce the theorem to these cases. In particular this includes the intersection dimension 4 case, which was claimed without proof in [**7**].

THEOREM. *Suppose M and X are proper submanifolds of Y, X has a normal microbundle ξ, there are closed subsets $C \subset D \subset Y$, and M is transverse to ξ near C. Then there is an isotopy of M supported in any given neighborhood of $(D - C) \cap M \cap X$, to a submanifold transverse to ξ near D.*

A submanifold is "proper" if closed, and the intersection with the boundary is the boundary of the submanifold. Transversality to a microbundle ξ means that the intersection $M \cap X$ is a manifold with a normal microbundle in M, and this microbundle is the restriction of ξ (see [**4**, III §1]). It is necessary to specify a bundle because the theorem is false for purely local versions of transversality; see [**4**, III §1]. Microbundle transversality implies transversality with respect to other bundle theories, e.g., [**6**].

Kirby and Siebenmann [**4**, p. 91] proved this assuming $\dim M \neq 4 \neq \dim M \cap X$, and either $\dim X \neq 4 \neq \dim Y$ or $\operatorname{codim} M \geq 3$. [**7**, Theorem 2.4.1] gave the remaining cases except when $\dim Y = 4$ and one of M, X has dimension greater than 2.

Received by the editors November 5, 1987.
1980 *Mathematics Subject Classification*. Primary 57N75.