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# CRABGRASS, MEASLES, AND GYPSY MOTHS: AN INTRODUCTION TO MODERN PROBABILITY

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This paper is based on a talk given at the Annual Meeting of the American Mathematical Society in San Antonio, Texas, January 21–24, 1978. The subject is interacting particle systems and the aim of the paper, like that of the talk, is to explain some of the results in this area to someone with no knowledge of probability theory except for an understanding of what it means to flip a coin with probability $p$ of heads. More than this is needed for some of the proofs given below, so a short appendix (three paragraphs) is provided which explains some of the concepts which may be unfamiliar.

In what follows we will discuss five models. The last three are hinted at in the title. The first two (Richardson's model and percolation) are related systems which are of interest in their own right, and will set the stage for explaining the other processes. Along the way the reader will encounter first-passage percolation, the subadditive ergodic theorem, branching processes, and large deviations, and will see how interacting particle systems can be used to study nonlinear PDE's.

**1. Richardson's model.** In this model the state at time $n$ is $\xi_n \subset Z^d$. When considering this process as a model of the spread of a biological population, we think of the points of $\xi_n$ as being "occupied". At other times when we think of Richardson's model as describing the spread of an infection through an orchard of trees, we will call the points in $\xi_n$ "infected". Both interpretations are common in the literature and we will use both below as convenience dictates. Using the first interpretation, the evolution of the process may be described as follows:

If $x \in \xi_n$ then $x \in \xi_{n+1}$.

If $x \notin \xi_n$ then $\mathsf{P}(x \notin \xi_{n+1} | \xi_n) = (1-p)^{\#\text{ of occupied neighbors}}$.

The first rule says there are no deaths. To explain the second rule we begin with the left-hand side. It says: "The probability that $x$ is not in $\xi_{n+1}$ given that $\xi_n$ is the state at time $n$." On the right-hand side, the neighbors of $x$ are the $2d$ points with $||x-y||_1 = 1$ (where $||x-y||_1 = |x_1-y_1| + \cdots + |x_d - y_d|$). In words, the rule says each occupied neighbor independently sends a particle to $x$ with probability $p$, so the probability they all fail to put a particle at $x$ is given by the right-hand side. The reader should note that the state at time $n$ is a subset of $Z^d$, i.e. each site is occupied by 1 or 0 particles, so if two neighbors simultaneously make the site occupied only one particle results.

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