it is a pity that the reprint apparently has not been used to make corrections, because there are many minor, but sometimes irritating, misprints.

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J. J. DUISTERMAAT

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Semigroups of linear operators and applications, by Jerome A. Goldstein. Oxford Mathematical Monographs, Oxford University Press, Clarendon Press, New York and Oxford, 1985, $x + 245$ pp., $42.50. ISBN 0-19-503540-2

This year is the centenary of the founding of the analytic theory of one-parameter semigroups: In 1887 Giuseppe Peano [**Pe**] wrote the system of ordinary differential equations

$$\begin{aligned} du_1/dt &= a_{11}u_1 + \cdots + a_{1n}u_n + f_1(t) \\ &\vdots \\ du_n/dt &= a_{n1}u_1 + \cdots + a_{nn}u_n + f_n(t) \end{aligned}$$

in matrix form as $du/dt = Au + f$ and found the explicit formula

$$u(t) = e^{tA}u(0) + \int_0^t ds\, e^{(t-s)A} f(s)$$

for the solution, where $e^{tA} = \sum_{k=0}^{\infty} t^k A^k / k!$. The mapping $t \geq 0 \to T(t) = e^{tA}$ is called the semigroup generated by A. More generally a C_0-semigroup on a Banach space $\mathbf{X}$ is a strongly continuous mapping from $\mathbf{R}_+$ into the bounded operators on $\mathbf{X}$ with the properties $T(t+s) = T(t)T(s)$ and $T(0) = 1$. The generator of T is the operator A defined by $Af = \lim_{t\to\infty}(T_t f - f)/t$ where f is in the domain $D(A)$ of A if and only if the limit exists. These concepts were introduced by Hille in the thirties, and he studied the semigroup by means of the resolvent $(\lambda - A)^{-1} = \int_0^\infty dt\, e^{-\lambda t} T(t)$. A fundamental theorem, proved by Hille and Yosida for contraction semigroups, and Feller, Miyadera, and Phillips for general semigroups around 1950, states that A is the generator of a C_0-semigroup T if and only if A is a closed, densely defined operator, and there exist real constants M, ω such that the resolvent $(\lambda - A)^{-1}$ exists for $\lambda > \omega$ and

$$\|(\lambda - \omega)^n (\lambda - A)^{-n}\| \leq M$$

whenever $\lambda > \omega$ and $n = 1, 2, 3, \ldots$. In this case $\|T(t)\| \leq Me^{\omega t}$, and

$$T(t)f = \lim_{n\to\infty} \left(1 - \frac{t}{n}A\right)^{-n} f \quad \text{for all } f \in \mathbf{X}.$$