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Geometric theory of foliations, by César Camacho and Alcides Lins Neto. Translated from Portuguese by Sue E. Goodman. Birkhäuser, Boston, Basel and Stuttgart, 1985, 205 pp., $42.00. ISBN 0-8176-3139-9

The word "foliation" stands out in the title of the book under review as requiring explanation. For those who are predisposed to botanical terminology (this definitely includes the reviewer) this term conjures up images which are perhaps reminiscent of Desargues [**20**]. The term "structure feuilletée" was coined by Georges Reeb [**2, 16**]. Later authors began using the briefer "feuilletage" which was translated as "foliation." Reeb winces at the botanical interpretation and offers instead a gastronomic motivation [**18**]:

> "A défaut de botaniste l'assemblée comptera peut-être un pâtissier. La pâte feuilletée—j'ai de bonnes raisons de le croire—donne une bonne idée d'un feuilletage (de codimension 1 dans R^3) dont elle dessine bien les feuilles et en suggère des propriétés."

Of course the more mundane translation to "sheeted" or "layered" shows that the terminology is appropriate since it suggests the image of pages in a book. Unlike the ill-fated terminology of Desargues, "foliation" has become a mathematical household word.

A foliation is simply a decomposition of a manifold into a disjoint union of immersed submanifolds (called leaves) of constant dimension such that the decomposition is locally homeomorphic to the decomposition of $R^n = R^k \times R^{n-k}$ into the parallel submanifolds $R^k \times \{\text{point}\}$. The most classical version of this is the "flow box" neighborhood of a point at which a vector field is nonzero ($k = 1$). With this case in mind the study of foliations may be thought of as a generalization of Poincaré's study of differential equations from the dynamic viewpoint [**14**]. One of the charming aspects of this subject is that there are several sources of historical motivation. Reeb, for example, derives inspiration from the study of differential equations in the complex domain inaugurated by Painlevé. Differential equations is not the only field which can claim the study of foliations. Geometers and topologists can also participate and claim antecedents such as É. Cartan, C. Ehresmann, H. Hopf, and H. Kneser.

The book under review deals with the principal results obtained in the theory of foliations during the period 1947–1965. The earliest of these results were the discovery of the Reeb foliation and the Reeb stability theorems [**17**]. H. Hopf had asked whether there is a completely integrable plane field on the 3-sphere. Reeb answered this question by constructing the now well-known foliation having a single toral leaf with all other leaves planar and spiralling towards the toral leaf. This example provided the justification for the further qualitative study of foliations. Furthermore, its significance in motivating later work of Haefliger and Novikov cannot be overstated.