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Let $\overline{\mathbf{R}}^n = \mathbf{R}^n \cup \{\infty\}$ be the one-point compactification of $\mathbf{R}^n$, $n \geq 1$. The group G_n of Möbius transformations is the transformation group on $\overline{\mathbf{R}}^n$ generated by the translations

$$x \mapsto x + a, \qquad a \in \mathbf{R}^n, \tag{1}$$

and the inversion

$$x \mapsto x/|x|^2 \tag{2}$$

in the unit sphere. There are a number of reasons why Möbius transformations play a central role in the geometry of $\mathbf{R}^n$. For instance:

(a) According to a classical theorem of Liouville, if $n \geq 3$, every conformal map from one subregion of $\mathbf{R}^n$ to another is the restriction of a Möbius transformation.

(b) The sense-preserving transformations in G_2 are the fractional linear transformations

$$g(z) = (az + b)(cz + d)^{-1}, \qquad ad - bc = 1, \tag{3}$$

which are fundamental tools in geometric function theory.

(c) If we embed $\mathbf{R}^n$ in $\mathbf{R}^{n+1}$ in the usual way, by identifying $\mathbf{R}^n$ with $(e_{n+1})^{\perp}$, formulas (1) and (2) define an action of G_n on $\overline{\mathbf{R}}^{n+1}$. In fact G_n is the subgroup of G_{n+1} that maps the half-space

$$H^{n+1} = \{x \in \mathbf{R}^{n+1}; x \cdot e_{n+1} > 0\}$$

onto itself. H^{n+1} with the Poincaré metric $ds = |dx|/(x \cdot e_{n+1})$ is the $(n+1)$-dimensional hyperbolic space, and G_n is its isometry group.

(d) Every Riemannian manifold of constant negative curvature (-1) can be represented as the quotient of H^{n+1} by a discrete subgroup Γ of G_n. In particular, the classical uniformization theorem implies that almost all