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ON FOURIER COEFFICIENTS OF A CONTINUOUS PERIODIC FUNCTION OF BOUNDED ENTROPY NORM

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In 1983 B. Korenblum [**5**, **6**] introduced a new class of Banach function spaces associated with the notion of entropy (we will call these spaces and their norms entropy spaces and entropy norms, respectively). The entropy norms are intermediate between the uniform and the variation norms. One application of entropy spaces is a new convergence test for Fourier series which includes classical tests of Dirichlet-Jordan and Dini-Lipschitz [**6**]. The aim of this note is to point out a natural connection between entropy spaces and Hardy space $\operatorname{Re} H^1$ [**4**]. In fact any entropy space can be embedded into $\operatorname{Re} H^1$ via a multiplier type bounded operator. As a corollary we obtain a growth condition for Fourier coefficients of a continuous periodic function of bounded entropy norm.

1. Two representations of an entropy norm. Let $T = R/Z$ be the unit circle, and let $|E| = \int_E dx$ denote the normalized Lebesgue measure of a Borel subset E of T. Also, let $k(s)$, $0 < s \leq 1$, be a positive nondecreasing concave function such that $k(s) = 1$. The k-entropy of a finite subset E of T ($E \neq \varnothing$) is $k(E) = \sum_i k(|I_i|)$, where $\{I_i\}$ are the complementary arcs of E in T. For an infinite subset E of T we set $k(E) = \sup\{k(F); F \subset E, F \text{ finite}\}$. We also put $k(\varnothing) = 0$. The *k-entropy norm* [**5**, **6**] of a real continuous function f on T is defined by the formula

$$\|f\|_k = \int_R k(f^{-1}(\{y\}))\, dy.$$

THEOREM 1 [**2**]. *Let $k(s)$, $0 < s \leq 1$, be a positive nondecreasing concave function such that $k(0^+) = 0$ and $\lim_{s\to 0}(k(s)/s) = \infty$. There is a unique Borel probability measure μ_k on the unit interval $(0,1]$ such that*

$$\|f\|_k = \int_T \int_{(0,1]} \frac{1}{s}\Omega_I(f)\, dx\, d\mu_k(s),$$

where $\Omega_I(f)$ is the oscillation of the function f on the arc $I = I(x,s)$ in T of length s and center at x. A relationship between k and μ_k is given by the formula

$$k(s) = \int_0^s \int_t^1 \frac{1}{u}\, d\mu_k(u)\, dt.$$

It is then proved in [**2**] that the set C_k of real continuous functions on T of finite k-entropy norm is a Banach algebra with respect to the norm $\|\cdot\|_k + \|\cdot\|_\infty$.

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