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ALGEBRAIC SURFACES AND 4-MANIFOLDS: SOME CONJECTURES AND SPECULATIONS

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Introduction. Since the time of Riemann, there has been a close interplay between the study of the geometry of complex algebraic curves (or, equivalently, compact Riemann surfaces) and the topology of 2-manifolds. These connections arise from the uniformization theorem, which asserts that every simply connected Riemann surface is conformally equivalent to either the Riemann sphere, the plane, or the interior of the unit disk. From this it follows that every compact Riemann surface has a conformally equivalent metric of constant curvature. A key idea in the proof of this result is the Dirichlet problem: Find a harmonic function on a Jordan region R in the plane with given boundary values. Any such harmonic function minimizes the functional

$$f \mapsto \iint_R |\nabla f|^2 \, dx \, dy$$

among all functions on R with the given boundary values. The existence of such functions has a physical interpretation. If we view the boundary values as a charge density on ∂R then the harmonic function corresponds to the resulting electrostatic potential in R. This physical interpretation suggests that such a harmonic function should exist and should be unique, at least for reasonable regions and boundary conditions. In fact one can solve this problem by integrating over the boundary a family of Green's functions, each of which is the electrostatic potential of a point charge, against the charge density.

The connection between the analysis and differential geometry of 2-dimensional metrics of constant curvature on the one hand and the topology and algebraic geometry of compact Riemann surfaces on the other has been a fruitful one. Major ideas have evolved from the work of Teichmüller [**39**], and Ahlfors and Bers [**1**, **6**] and more recently through the work of Thurston [**40**]. Some of the nicest recent examples of this interplay can be found in [**20**], with applications to the algebraic geometry in [**21**].

Of course a smooth complex algebraic variety of dimension n is naturally a C^∞-manifold of dimension $2n$. But for $n \geq 3$ the algebraic geometry of these varieties diverges quite markedly from the topology for many reasons, some of which will be made more precise below. Quite surprisingly, very deep connections have emerged recently between the complex geometry of a complex

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