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Vanishing theorems on complex manifolds, by Bernard Shiffman and Andrew John Sommese, Progress in Mathematics, vol. 56, Birkhäuser, Boston, Basel, Stuttgart, 1985, xiii + 170 pp., $19.95. ISBN 0-8176-3288-3

On a topological or differentiable manifold local objects (functions, vector fields, differential forms, ...) can be extended to global ones by a partition of unity. On complex or algebraic manifolds this is generally not possible. The obstructions for doing this lie in a (first) cohomology group. Therefore it is of prime interest to know conditions guaranteeing the vanishing of such cohomology groups.

If the manifold is Stein or affine this is always the case but for compact complex manifolds the situation is completely different. In the compact case, however, the cohomology groups $H^q(X, F)$ are always finite dimensional $\mathbf{C}$-vector spaces (F being a coherent $\mathcal{O}_X$-module sheaf). Under the heading "vanishing theorems" we understand general statements when such groups do vanish, however. Here F mostly is a holomorphic line bundle or, more generally, a vector bundle of higher rank.

The most famous vanishing theorem is the one of Kodaira proved in 1953: *Let L be a positive holomorphic line bundle on the compact complex manifold X. Then*

$$H^q(X, L \otimes K_X) = 0 \quad \text{for } q \geqslant 1.$$

Here K_X denotes the canonical line bundle (i.e. K_X is the line bundle of holomorphic n-forms, $n = \dim_{\mathbf{C}} X$). The positivity of a line bundle can be defined in several ways:

(i) *differential-geometric*: L admits a hermitian metric h such that the curvature form $\theta_h = i\bar{\partial}\partial \log h$ is a positive (1, 1)-form.

(ii) *function-theoretic*: the zero-section of the dual bundle L^{-1} admits a strongly pseudoconvex neighborhood.

(iii) *algebraic-geometric*: the sections of a high-power L^m embed X into a projective space.

(iv) *numerical*: $c_1(L \mid Y)^s > 0$ for all irreducible reduced analytic subspaces $Y \subset X$ of dimension s (here c_1 denotes the first Chern class).

The equivalence of these conditions is by no means obvious. For instance the equivalence of (i) and (iii) is the celebrated Kodaira embedding theorem and (iv) is the Nakai-Moishezon criterion.