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SMOOTH NONTRIVIAL 4-DIMENSIONAL s-COBORDISMS

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ABSTRACT. This announcement exhibits smooth 4-dimensional manifold triads $(W; M_0, M_1)$ which are s-cobordisms, i.e. the inclusions $M_i \subseteq W$, $i = 0, 1$, are simple homotopy equivalences, but are not diffeomorphic or even homeomorphic to a product $M_i \times [0, 1]$.

The Barden-Mazur-Stallings s-cobordism theorem constitutes one of the foundational stones of modern topology. It asserts, in the smooth, piecewise-linear, or topological categories, that if W is a manifold of dimension at least six, with boundary components M_i, $i = 0, 1$, whose inclusions into W are simple homotopy equivalences, then W is necessarily a product (see [**K**, **H**, **RS**, **KS**]). For simply connected smooth manifolds of dimension at least six, this result had already been proven by Smale as the "h-cobordism theorem" [**Sm**], with the generalized Poincaré conjecture in higher dimensions as a corollary. The s-cobordism statement holds in dimensions one and two, and is equivalent to the Poincaré conjecture in dimension three. Freedman [**F1**, **F2**] proved the five-dimensional result for topological manifolds with fundamental group of polynomial growth (e.g. finite or polycyclic). Donaldson's extraordinary results imply the failure of the five-dimensional result in the smooth (or piecewise linear) category even for simply connected manifolds; by [**F1**] the resulting h-cobordisms will still be topological products. Using Freedman's results, the present authors produced some nontrivial orientable four-dimensional topological s-cobordisms [**CS1**, **CS2**]. (See [**MS**] for a nonorientable and definitely nonsmoothable example.) These topological constructions have been further studied and extended by Kwasik and Schultz [**KwS**].

We will now use a different construction to produce some nontrivial smooth s-cobordisms. Neither the construction nor the proof rely on any of the results cited above. Let M be a quaternionic space-form; i.e.

$$M = M_r = S^3/Q_r,$$

Q_r the quaternionic group of order 2^{r+2}. Then it is well known that the orientable manifold M has a one-sided Heegaard splitting

$$M = N(K) \cup H,$$

where $N(K)$ is the total space of an interval bundle over the Klein bottle K and H is a solid torus. Let E_0 be a closed tubular neighborhood of

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