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ISOSPECTRAL RIEMANNIAN METRICS AND POTENTIALS

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For (M, g) a compact Riemannian manifold, we consider the spectra of the Laplace-Beltrami operator and of the Schrödinger operator "Laplacian plus potential" acting on $L^2(M, g)$. Two Riemannian manifolds are said to be isospectral if their associated Laplacians have the same spectra, and two potentials on the same Riemannian manifold are said to be isospectral if the associated Schrödinger operators have the same spectra. Generalizing methods of Sunada [**S**] and Brooks [**B**], we give a fairly general technique for constructing isospectral metrics and potentials. In the case of metrics, our method unifies the various methods used previously by numerous authors to construct isospectral metrics and allows us to construct many new examples; in the case of potentials, we obtain many new examples of continuous families of isospectral, noncongruent potentials.

In all the examples known of isospectral closed manifolds, the manifolds have a common Riemannian cover. Thus, they are of the form $(\Gamma_i \backslash M, g)$, $i = 1, 2$, where (M, g) is a Riemannian manifold and each Γ_i is a discrete group acting freely and properly discontinuously by isometries on (M, g). Moreover, with one exception (noted below), there exists a bijection between Γ_1 and Γ_2 such that corresponding elements are conjugate in the full isometry group $I(M)$. It is not known whether these conditions are sufficient for the quotient manifolds to be isospectral. Sunada [**S**] proved under these conditions that if Γ_1 and Γ_2 are both contained in a finite subgroup G of $I(M)$ which acts freely on M and if corresponding elements of Γ_1 and Γ_2 are conjugate within G, then the manifolds are isospectral. (Of course, if the groups Γ_1 and Γ_2 are conjugate, then the manifolds are isometric.)

Before stating our first theorem, we recall that any Lie group G which admits a uniform discrete subgroup is unimodular. For $\gamma \in \Gamma$, the centralizer $C(\gamma, \Gamma)$ is a uniform discrete subgroup of the centralizer $C(\gamma, G)$, so $C(\gamma, G)$ is also unimodular. Given any conjugacy class of unimodular subgroups of G, we can define a Haar measure on each subgroup in such a way that if $K = aHa^{-1}$, then the conjugation by a is a measure-preserving transformation from H to K. In the following theorem, we assume Haar measures have been so chosen on $C(\gamma_i, G)$. For any group H and any element $h \in H$, we denote by $[h]_H$ the conjugacy class of h in H, and we denote by $[H]$ the set of all conjugacy classes of elements of H, so $[h]_H \in [H]$.

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