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An introduction to chaotic dynamical systems, by Robert L. Devaney, Benjamin/Cummings Publishing Company, Menlo Park, 1986, xii + 320 pp. ISBN 0-8053-1601-9

Normally, the Bulletin of the American Mathematical Society reviews research monographs and graduate texts. Rarely does it seem suitable that more elementary books be reviewed. In my opinion, the editors rightly felt that the book by Devaney is such an exception. The undergraduate curriculum in mathematics evolves rather slowly. Infrequently a serious attempt is made to introduce modern mathematics into the undergraduate curriculum. The reasons are clear. Most modern mathematics requires sufficient background preparation that it is simply not accessible to a typical undergraduate. Moreover, there is a question of priorities in choosing topics courses that are suitable for mathematics majors. Devaney has written a book which deserves serious consideration as a suitable undergraduate text for an advanced undergraduate topics course.

The main subject of Devaney's book, iteration of functions in one real and complex dimension, has grown to maturity in the past decade from humble and largely forgotten origins. It has flowered into an object of considerable mathematical (and visual) beauty and subtlety. Still, the basic phenomena of the real theory can be explained and understood with little background beyond the standard fare of calculus, linear algebra, and basic complex analysis. Additionally, the subject is one that is easy to illustrate, and it provides a wonderful arena for exploration with a computer. As a potential new item in the undergraduate curriculum, the importance of the subject and particularly the style of thought deserve extended discussion.

Iteration is a process that is frequently encountered in both the natural world and in artificial ones. An important example in an artificial world is to describe the behavior of Newton's method applied to a fixed function. If we believe that there are hard and fixed rules that determine the evolution of economies, populations, planets, or blocks sliding on inclined planes, then we confront the mathematical problem of making long-term predictions from our knowledge of how the rules operate for fixed short times. For continuous time, this is the problem of solving systems of ordinary differential equations. The usual undergraduate courses in differential equations begin with classical techniques for finding explicit solutions to systems of ordinary differential equations. The material has the flavor of "techniques of integration" in calculus and often is presented as a collection of tricks. One of the facts of life is that most differential equations do not have explicit analytic integrals, so the techniques one learns for solving equations explicitly are limited. Indeed, the presence of "chaos" in solutions is often an obstruction to the existence of analytic integrals. Dynamical systems theory seeks qualitative information about the solutions of general equations, often in a manner that looks at all of the phase space. This is an important problem that pervades much of science.