

The book under review, which is the outgrowth of a series of introductory lectures, is written on two levels. The initial portion of each chapter deals with the shift operator of unit multiplicity and delves quite deeply into the associated function theory, paying special attention to the spectral analysis of the parts of the backward shift operator. This material could serve as an introduction to shift-related operator theory and function theory for someone with a basic background in functional analysis and complex analysis. All but a few chapters contain supplementary sections where the earlier material is refined and extended; in particular, multiple shift operators are studied. The style here becomes that of an advanced monograph. Besides the eleven main chapters there are five appendices, themselves comprising about 45 percent of the text. One, of 100 pages, gives an introduction to the spectral theory of two kinds of operators closely related to the shift operator, Hankel and Toeplitz operators; another, of 56 pages and contributed by S. V. Hruščev and V. V. Peller, further develops the theory of Hankel operators, especially the connections of those operators with approximation problems and with stationary Gaussian sequences.

This is a book for the devotee, or the would-be devotee. If my experience is typical, those who love the subject will love the book.

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 16, Number 2, April 1987
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0273-0979/87 \$1.00 + \$.25 per page

An introduction to nonstandard real analysis, by A. E. Hurd and P. A. Loeb, Pure and Applied Mathematics, vol. 118, Academic Press, 1985, xii + 232 pp., \$35.00. ISBN 0-12-362440-1

Nonstandard analysis is now widely applied in a number of different mathematical fields. A partial list of the applications includes functional analysis (Bernstein and Robinson [15], and the survey by Henson and Moore [21]), perturbation theory (Lutz and Goze [38]), mathematical physics (Arkeryd [9, 10, 11, 12, 13]), potential theory (Loeb [37]), mathematical economics (Brown and Robinson [16, 17], Anderson [4, 6, 8], the references in [7], and the forthcoming book by Rashid [46]), and probability theory (see the survey by Cutland [18]).

Standard mathematicians tend to test the worth of nonstandard analysis by asking whether it has led to new standard results in their fields. It is not clear that this is the right test: after all, most fields yield far more results of internal interest than applications to other fields. Nonetheless, it is a test which nonstandard analysis is beginning to meet.

In most of the above areas, nonstandard analysis has led to new standard theorems. A metatheorem guarantees that any standard theorem provable by nonstandard methods has a standard proof; this is important, since it tells us that any theorem with a nonstandard proof follows from the usual axioms of