

11. R. V. Moody, *Lie algebras associated with generalized Cartan matrices*, Bull. Amer. Math. Soc. **73** (1967), 217–230.
12. ———, *A new class of Lie algebras*, J. Algebra **10** (1968), 211–230.
13. ———, *Euclidean Lie algebras*, Canad. J. Math. **21** (1969), 1432–1454.
14. J.-P. Serre, *Algèbres de Lie semi-simples complexes*, Benjamin, New York and Amsterdam, 1966.
15. I. Singer and S. Sternberg, *On the infinite groups of Lie and Cartan*, J. Analyse Math. **15** (1965), 1–114.
16. G. M. Benkart, *A Kac-Moody bibliography and some related references*, Lie Algebras and Related Topics (D. J. Britten, F. W. Lemire, and R. V. Moody, eds.), Canadian Math. Soc. Conf. Proc. vol. 5, 1984, pp. 111–135, Amer. Math. Soc., Providence, R. I., 1986.
17. I. B. Frenkel, *Representations of affine Lie algebras, Hecke modular forms and Korteweg-deVries type equations*, Lie Algebras and Related Topics (D. J. Winter, ed.), Lecture Notes in Math., vol. 933, 1981, pp. 71–110. Springer-Verlag, Berlin-Heidelberg-New York, 1982.
18. G. Segal and G. Wilson, *Loop groups and equations of KdV type*, Publ. Math. Inst. Hautes Études Sci. **61** (1985), 5–65.

GEORGE B. SELIGMAN

BULLETIN (New Series) OF THE  
 AMERICAN MATHEMATICAL SOCIETY  
 Volume 16, Number 1, January 1987  
 ©1987 American Mathematical Society  
 0273-0979/87 \$1.00 + \$.25 per page

*Hardy classes and operator theory*, by Marvin Rosenblum and James Rovnyak, Oxford Mathematical Monographs, Oxford Univ. Press, New York and Clarendon Press, Oxford, 1985, xii + 161 pp., \$39.95. ISBN 0-19-503591-7.

Hardy space theory has its classical origins in the work of G. H. Hardy and the brothers Riesz, but the modern origins of the subject begin with the theorem of A. Beurling in 1949. The Hardy space  $H^2$  is defined to be the space of functions  $f$  analytic on the unit disk such that

$$\|f\|_2^2 = \sup \left\{ \frac{1}{2\pi} \int_0^{2\pi} |f(re^{it})|^2 dt : 0 \leq r < 1 \right\} < \infty.$$

The theorem of Beurling asserts that any such  $f$  has an inner-outer factorization  $f = bg$  where  $b$  is an inner function and  $g$  is an outer function. By definition an inner function is a function analytic on the unit disk whose nontangential boundary values have modulus 1 almost everywhere on the unit circle. An outer function can be defined as the solution of the extremal problem of finding the function  $g$  in  $H^2$  that maximizes  $|g(0)|$  among all functions with  $|g(e^{it})|$  equal to a prescribed function on the boundary. Both inner functions and outer functions have finer structure; an inner function can be factored further as the product of a Blaschke product and a singular inner function while an outer function is characterized by having an integral representation of a certain form. It was recognized already by Beurling that this purely function-theoretic result has connections with operator theory. Indeed, from this theorem one can classify all the closed invariant subspaces for the