BOOK REVIEWS

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User's guide to spectral sequences, by John McCleary, Mathematics Lecture Series No. 12, Publish or Perish, Inc., Wilmington, Delaware, 1985, xiii + 423 pp., $40.00. ISBN 0-914098-21-7

What are spectral sequences, and what are they good for? Perhaps we can best answer this question by giving a couple of examples.

First example, from homological algebra. Let G be an arbitrary group, and let A be a G-module. To understand this example, the reader needs to know a little bit about the cohomology groups of G with coefficient module A, denoted by $H^q(G, A)$, $q = 0, 1, 2, \ldots$. In the study of these cohomology groups the following problem arises: Let N be a normal subgroup of G; assume we know the cohomology groups of the subgroup N and the quotient group G/N (with some suitable choices of coefficient module). With this information, can we determine the cohomology groups of G? A little thought should convince the reader that, in general, the answer will probably be No, because usually there exist many different, nonisomorphic "extensions" G, given N and G/N. In other words, more information will be needed.

Thus a more reasonable problem is the following: Determine what relations must exist between the cohomology groups of N, G/N, and G. The answer to this problem is given by a spectral sequence. We will now explain what this spectral sequence is.

The reader will recall that the cohomology groups of G are defined by means of a certain "cochain complex," which consists of a sequence of abelian groups $\{C^q \mid q = 0, 1, 2, \ldots\}$ together with a "coboundary operator," δ, which is a homomorphism $\delta: C^q \to C^{q+1}$ defined for all $q \geqslant 0$ and having the basic property that $\delta \circ \delta = 0$. The subgroup of C^q which is the kernel of δ is denoted by Z^q, and the image subgroup $\delta(C^{q-1})$ is denoted by B^q. From the basic property $\delta \circ \delta = 0$, it follows that $B^q \subset Z^q$; and the quotient group, Z^q/B^q, is by definition the qth cohomology group, H^q.

In connection with spectral sequences the following alternative terminology for these concepts has become common: the sequence of abelian groups $\{C^q\}$ is called a "graded abelian group", and the homomorphism δ is said to have "degree $+1$", because $\delta(C^q) \subset C^{q+1}$, and to be a "differential" because $\delta \circ \delta = 0$. The following slight generalization of these concepts has also become standard: a *bigraded* group is a doubly indexed sequence of abelian groups,

$$E = \{E^{p,q}\}$$