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MONOPOLES ON ASYMPTOTICALLY EUCLIDEAN 3-MANIFOLDS

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ABSTRACT. We consider a generalization of Yang-Mills-Higgs theory on Euclidean $\mathbf{R}^3$ to connected sums of $\mathbf{R}^3$ with compact closed 3-manifolds.

In this note, we describe progress in Yang-Mills-Higgs theory on 3-dimensional Riemannian manifolds. In particular, we are interested in the set of minima of the Yang-Mills action

$$\mathfrak{A}(A,\Phi) = \int_M (|F_A(x)|^2 + |\nabla_A \Phi(x)|^2)\, d\mu(x), \tag{1}$$

where the "Higgs field" Φ is a section of a metric vector bundle E over M and A is a linear connection on E preserving the metric. For simplicity, we will restrict ourselves to the case where $E = \mathrm{ad}(P)$ is the adjoint bundle of an SU_2-principal bundle P over M.

The functional $\mathfrak{A}$ has been studied in great detail in the case where M is the Euclidean $\mathbf{R}^3$, see [**8**]. We recall here briefly the main results: The length of the Higgs field Φ of any finite action configuration $c = (A,\Phi)$ obtains in some sense an asymptotic value $m(c)$ at infinity. For each $m > 0$, the space of finite action configurations c with $m(c) = m$ decomposes into a family of components indexed by a "topological charge" k. On each of these components, the minima of the action functional (1) can be shown to be solutions of the Bogomolny equation

$$\mathfrak{b}_\pm(c) = \nabla_A \Phi \mp {}^*F_A = 0 \tag{2}$$

with the sign equal to the sign of k. Since reversing the sign of Φ changes the sign of k while leaving $\mathfrak{A}$ invariant, we can restrict ourselves to the case $k > 0$ and write $\mathfrak{b} = \mathfrak{b}_+$. The set of gauge equivalence classes of solutions of (2), also called monopoles, is a smooth manifold $\mathcal{M}_k$ of dimension $4k$. It can be described by means of algebraic geometry, see [**7** and **3**]. One usually considers the $(4k-1)$-dimensional submanifolds $\mathcal{M}_k^m$ of monopoles $[c]$ with fixed "mass" $m(c) = m$. In fact, one can without loss of generality set $m = 1$, since a scaling involving a dilation of $\mathbf{R}^3$ shows that $\mathcal{M}_k^m \cong \mathcal{M}_k^1$ for all $m > 0$.

In order to generalize these ideas, we replace $\mathbf{R}^3$ by an asymptotically flat manifold M, see [**9**]. This means that M is the connected sum of $\mathbf{R}^3$ with a compact manifold, equipped with a metric which at the end of M is a perturbation of the Euclidean metric in a certain sense. Since in this

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