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ON THE LIE SUBGROUPS OF INFINITE DIMENSIONAL LIE GROUPS

J. LESLIE

1. Introduction. Milnor [**4**] posed the question, "Does every closed Lie subalgebra of the Lie algebra of an infinite-dimensional Lie group modelled on a complete locally convex topological vector space correspond to an immersed Lie subgroup?" Omori [**5**] has shown that the answer to this question is negative in general; in this note we outline conditions under which a positive answer can be given.

Correspondence with Milnor has been helpful in preparing the present version of this research announcement.

All of the theorems in this paper have proofs which are written down. The author is in the process of preparing a manuscript in which detailed proofs are presented.

Let us recall that a subset of a real vector space E is said to absorb a subset B of E when there exists a constant $\lambda > 0$ so that $\lambda B \subseteq A$.

We shall call a subset, S, of a vector space which is circled (i.e. $|\lambda| \leq \underline{1}$ and $s \in S$ implies $\lambda s \in S$) and convex a disk.

DEFINITION 1. A bornological vector space is a Hausdorff, locally convex, topological vector space in which any disk which absorbs every bounded subset of E is a neighborhood of the origin.

EXAMPLE 1. A metrizable locally convex topological vector space is bornological [**1**].

EXAMPLE 2. The locally convex inductive limit of bornological spaces is bornological [**1**].

CONVENTION. C° will mean continuous in this paper.

DEFINITION 2. We say that $f\colon U \to F$ is Gateaux C^n smooth when there exists k-multilinear symmetric continuous functions $D^k f(x)\colon E\times \cdots \times E \to F$, $1 \leq k \leq n$, $x \in U$, so that each

$$D^k f\colon U \times E \times \cdots \times E \to F$$

is continuous and each

$$F_k(v) = f(x+v) - f(x) - Df(x)(v) - \cdots - \frac{1}{k!} D^k f(x)(v, \ldots, v), \qquad 1 \leq k \leq v,$$

satisfies the property that

$$G_k(t,v) = \begin{cases} F_k(tv)/t^k & t \neq 0, \\ 0, & t = 0, \end{cases}$$

is continuous at $(0, v)$.

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