# BOOK REVIEWS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 15, Number 2, October 1986

0273-0979/86 $1.00 + $.25 per page

*Non-archimedean analysis*, by S. Bosch, U. Güntzer, and R. Remmert, Grundlehren der mathematischen Wissenschaften, Vol. 261, Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1984, xii + 436 pp., $59.00. ISBN 3-540-12546-9

*Géométrie analytique rigide et applications*, by Jean Fresnel and Marius van der Put, Progress in Mathematics, Vol. 18, Birkhäuser, Boston, Basel, Stuttgart, 1981, xii + 215 pp., $17.50. ISBN 3-7643-3069-4

**1. Tate's rigid analytic geometry.** Both books under review deal with a theory which was created in 1961 by John Tate, who at that time had given a seminar on it at Harvard and written a manuscript entitled *Rigid analytic spaces*.

These notes by Tate were distributed in Paris by the IHES in the spring of 1962 with(out) his permission and published as late as 1971 in *Inventiones Mathematicae*, whose editors thought it necessary to make these available to everyone. It is strange that the man who created this beautiful theory did nearly nothing to make it known. Further, to my knowledge he has never taken up research on the foundations of rigid analytic geometry which he has laid. I cannot guess for what reason he did not like his child later on.

I will now try to give a very rough idea of what the subject is all about. Let $K$ be a field and $|\ |$ a *valuation* in $K$: a real-valued function on $K$ for which $|0| = 0$, $|1| = 1$, $|a \cdot b| = |a| \cdot |b|$, $|a + b| \leqslant |a| + |b|$ holds for any $a, b \in K$. These valuations were introduced in order to better understand the fields of $p$-adic numbers constructed by Kurt Hensel in 1905. It was again Hensel who first studied $p$-adic analytic functions in his book *Zahlentheorie* of 1913, where he investigated properties of the $p$-adic exponential and logarithm. If the field $K$ is complete with respect to the valuation, then it makes sense to single out the convergent power series. On any $K$-algebraic variety $V$ one has then a natural notion of analytic functions, namely those functions which have locally convergent power series expansions in algebraic parameters. This analytic structure on $V$ was studied in the Cartan seminar of 1960/1961. If the valuated field $K$ is different from the field of real or complex numbers, then it is nonarchimedean, which is equivalent to being ultrametric, that is, $|a + b| \leqslant \max\{|a|, |b|\}$ is satisfied for any $a, b \in K$. In this case $K$ is totally disconnected and the above-mentioned analytic structure fails to fulfill the basic principle of analytic continuation. As Tate says, the analytic structure gets wobbly. The main discovery of Tate consists in a procedure to define a rigid analytic structure which saves the principle of analytic continuation and gives a useful