BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 15, Number 2, October 1986

PROPAGATION OF SINGULARITIES OF SOLUTIONS TO SEMILINEAR BOUNDARY VALUE PROBLEMS

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ABSTRACT. Let P be a second-order, strictly hyperbolic differential operator on an open region $\Omega \subset \mathbf{R}^n$ $(n \geq 3)$ with smooth noncharacteristic boundary. Given a solution $u \in H^s_{\mathrm{loc}}(\overline{\Omega})$, $s > (n+1)/2$, to $Pu = f(x, u)$, we discuss the propagation of microlocal H^r singularities in the range $s \leq r < 2s - n/2$ in the general case where the Hamilton field of p may be tangent to $\partial T^*\Omega\backslash 0$ to arbitrarily high finite or infinite order.

Introduction. Recently, M. Sablé-Tougeron [**10**, **11**] and A. Alabidi [**1**] used the calculus of paradifferential operators to obtain, for a quite general class of nonlinear boundary value problems, results describing the reflection of singularities that travel on bicharacteristics transversal to the boundary of a region $\Omega \subset \mathbf{R}^n$. Using a different approach, similar to the one initiated by Rauch [**9**] and further developed by Beals and Reed [**3**] in studying interior propagation, we have obtained results for second-order semilinear problems with Dirichlet conditions in the general case where tangential bicharacteristics as well as gliding rays may carry singularities. Our argument has two main steps. We first prove an analogue of Rauch's Lemma for a class of spaces measuring microlocal H^s regularity up to the boundary. We also require a precise linear theorem (Theorem 2), a refinement of the results of Melrose and Sjöstrand ([**8**], see also [**6**]), which describes the propagation of H^s regularity along generalized bicharacteristics. This theorem must apply to equations $Pu = v$ for distributions v which lie in our microlocal algebras, but which cannot be assumed to be normally regular. A simple inductive argument combining the above then yields the desired semilinear theorem (Theorem 1). Complete proofs will appear in [**4**].

Spaces of distributions near $\partial\Omega$. Let $\tilde{U}$ be a coordinate chart centered at $x_0 \in \partial\Omega$, and let (x_1, x') be coordinates in which $\overline{\Omega} \cap \tilde{U}$ $(\equiv U) = \{x_1 \geq 0\}$. We will use the spaces $H^{\mathrm{loc}}_{t,t'}(U)$ defined in Hörmander [**5**]. For $u \in H^{\mathrm{loc}}_{r,-\infty}(U)$ we define $\widetilde{WF}_r(u) \subset (T^*\partial U\backslash 0) \cup (T^*\mathring{U}\backslash 0)$ as follows. If $\sigma \in T^*\partial U\backslash 0$, $\sigma \notin \widetilde{WF}_r u$ if and only if for some tangential pseudodifferential operator $\phi(x, D')$ of order zero, elliptic at $(0, \sigma)$, $\phi u \in H^r_{\mathrm{loc}}(U)$. In this case we write $u \in \tilde{H}^r(\sigma)$. In $T^*\mathring{U}$, $\widetilde{WF}_r$ coincides with the usual notion of WF_r. Note that the definition of $\widetilde{WF}_r u$ is coordinate dependent. However, it is a consequence (see [**4**]) of the Lemma stated below and the fact that $\partial\Omega$ is noncharacteristic, that for

Received by the editors December 10, 1985.
1980 *Mathematics Subject Classification*. Primary 58G17, 35L70.

0273-0979/86 $1.00 + $.25 per page